THE VIOLIN

MICHAEL MENDELSON
M.E. 199
PROF. CARROLL
JUNE, 1971

42 San Francisco Chronicle ☆☆ Fri., June 4, 1971

Strad Sold for $201,600

A Record Price for a Violin

By Harold C. Schonberg
N.Y. Times Service

London

A record auction price for a violin was established yesterday when the firm of W. E. Hill and sons paid $201,600 for a Stradivarius.

The previous record, also at a Sotheby's auction, had been set in 1968 for the Marie Hall Stradivarius, which fetched $53,000.

The violin sold yesterday is known as the lady blunt. It was made by Antonio Stradivari in 1721. In perfect condition, the instrument still has its original varnish, and there is not a crack in it. Yehudi Menuhin, who played it the other day, exclaimed over its tone and said the "marvelous" instrument has been "maintained in loving care through the years."

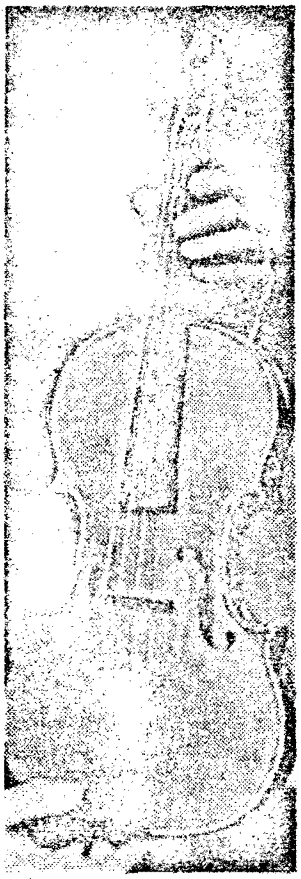
BIDDER

The successful bidder, Andrew Hill, purchased the instrument for a collector, but would not at this time divulge the name or nationality of the new owner.

The bidding started at $24,000. Within a few minutes only Hill and the firm of J. and A. Beare were in competition. At Hill's last bid there was a burst of applause in the crowded auction room.

SECRETS

Howard Ricketts, of the Sotheby firm, was the auctioneer, and his voice did not change expression when he closed out the bidding at $201,600.



UPI Telephoto

THE LADY BLUNT
Not a crack in it

Although the price of the Lady Blunt almost quadrupled the previous high actuion price for a violin, it is not known whether it established an all-time record for a sale. Private collectors have been known to pay enormous prices, but those sums are well-kept secrets in the trade. The consensus was, however, that the price for the Lady Blunt was the highest ever paid for any instrument.

As with als important Cremones instruments, the history of the Lady Blunt is well documented. In 1864 the famous French violin-marker J. B. Vuillaume located the instrument in Spain. In his own words, "It was brought to me in an unheard-of condition, with the neck, fingerboard and bass bar of Stradivari. It had never been opened, the reason being that it had reposed, forgotten, in an attic for over a hundred years."

MODIFICATIONS

Vuillaume, in accordance with modern practice, lengthened the neck and inserted a new fingerboard. He sold the instrument to Lady Ann Blunt the same year, for the then record price of $624. Lady Blunt, a granddaughter of Lord Byron, was a good violinist who had studied with Norma Neruda, perhaps the greatest woman violinist of all time.

At Lady Blunt's death the violin passed through a series of private collections. Its last owner was Sam Bloomfield, a collector in Los Angeles. Thus the Lady Blunt strad has never been used by a professional violinist, which accounts for its pristine shape. No instrument can take the abuse of daily concert work, and most of the great violinists throughout history have owned several ceremonese instruments, which they alternate. That remains true today.

In recent years doubt has been cast on the superiority of the instruments of Amati, Stradivari, Guarneri, Bergonzi and the other great violin makers, most of whom lived in Cremona. Scientists say they can measure no appreciable difference between a good modern instrument and a cremonese. Attempts have been made by researchers to isolate the secrets of the cremonese masters, if indeed there is a secret.

COMBINATION

But scientists notwithstanding, professional violinists say a Stradivarius or Guarnerius has a combination of sweetness and power that no modern instrument can come near matching. Hence the excitement when a great cremonese instrument becomes available.

Fewer and fewer can be obtained as museums and other institutions purchase them and take them permanently from the market.

The Messiah Stradivarius, for instance, which is considered to be the most perfect of all cremonese instruments, belongs to the Ashmolean museum in Oxford.

Many violinists attended the auction, and all expressed a hope that the Lady Blunt would be used in public rather than sit forever behind a glass case.

TABLE OF CONTENTS

INTRODUCTION

During the years 1838 and 1839, M.F. Savart gave a series of lectures in France on the properties of the violin. The lectures, " Analyses de Cours Scientifiques. Cours de Physique Experimentale, professe au College de France pedant l'annee scolaire 1838-1839, par M.F. Savart, professeur" later reprinted by L'Institute, Paris, France 1840, were the first serious attempts to apply a rational scientific method to the question of how a violin works, and what distinguishes a good violin from a bad one. Parts of these lectures are reprinted in Davidson's book, The Violin.[6] Savart went into the basic mechanical processes that produce the sound of the violin, conducting crude experiments to show the effects of the bridge, the bass bar, the sound post and other features of the instrument. Very little else was done on the subject until the advent of modern electronic sound analyzers.

In recent years, the analysis of the tonal properties of the violin has become considerably more refined. Sound intensity curves,* plotting loudness versus frequency have revealed the importance of resonances, both air resonances and body resonances, on the tone of an instrument. These detailed curves allow comparison on violins considered "good" by the experts, and reveal certain aspects that can be built into new violins such that the event of the building of an excellent instrument is no longer such a hit and miss proposition. The work of the late F.A. Saunders, and of others since then have found what appear to be some universal features of good violins in terms of those resonances, and have developed ways for reproducing those features.

In 1946, Saunders ran test on many fine violins. In

* SEE FIGURES 1-3-4-5-6

②

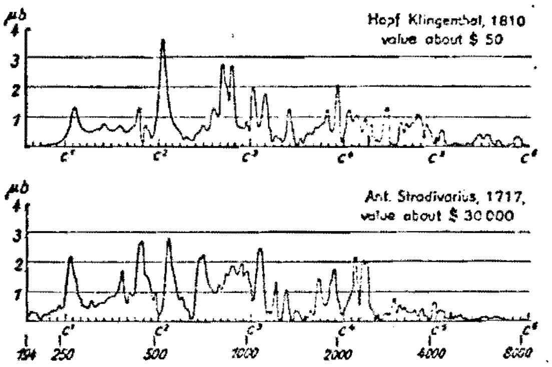


FIG. 1 Response curves of a Strad (a violin of very good tone quality) and a Hopf, Klingenthal (a violin of mediocre tone quality).

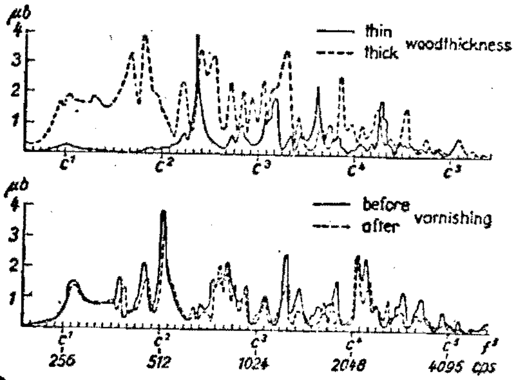


FIG. 2 The influence of wood-thickness is relatively great; the influence of varnish is comparatively small.

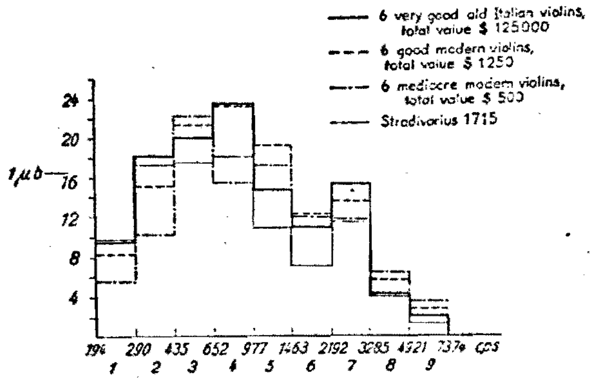


FIG. 3 Average response curves of groups of violins of different tone quality.

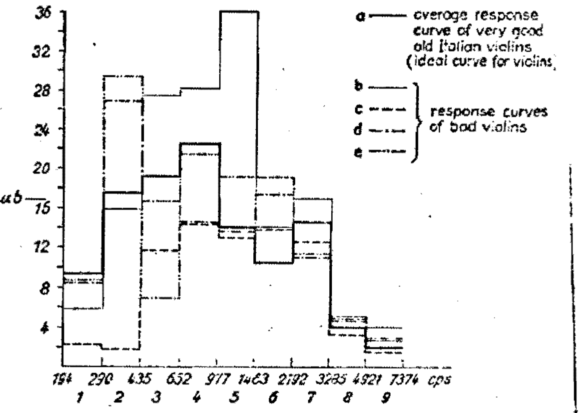


FIG. 4 Response curves of bad violins in comparison with th average response curve of very good old Italian violins.

TABLE IV. Intensities in different frequency ranges.

Violins	Method	I	II	III	IV	V	VI
Strads, by both methods							
Strad K (1684)	old	15.9	22.9	22.3	24.5	24.7	11.4
Strad A (1691)	old	15.5	23.7	24.1	21.9	25.7	15.7
Strad B (1694)	old	17.3	19.9	28.1	25.9	25.7	7.3
	new	15.2	25.0	24.6	26.5	20.8	7.1
Strad M (1698)	old	11.7	24.5	25.3	26.7	24.1	10.5
Strad J (1709?)	old	15.1	24.9	25.1	26.1	22.9	6.9
	new	11.9	22.3	24.7	25.3	24.5	16.4
Strad D (1712)	new	16.0	23.9	23.8	24.2	26.3	9.0
Strad E (1715)	old	15.3	24.1	25.7	24.3	24.7	9.1
	new	15.2	25.1	24.6	26.2	21.7	6.5
Strad W (1715)	old	12.5	23.3	23.9	27.3	24.5	12.9
	new	14.5	23.2	25.0	24.6	22.7	12.5
Strad S (1723)	old	13.1	23.9	24.6	26.3	25.3	12.3
	new	14.3	23.3	30.1	28.6	25.5	14.0
Strad H (1731)	old	14.9	25.1	23.7	26.5	19.7	10.0
Strad G (1732)	new	11.6	23.0	26.9	25.9	23.7	10.6
Strad N (1737)	old	10.1	25.7	24.7	27.3	21.5	11.3
Strad average		14.1	23.8	25.1	25.8	23.8	10.8
Guarnerius d. Gesu violins; old method							
Guarnerius R (1743)	old	10.3	25.7	24.5	26.3	25.9	10.3
Guarnerius H (1742)	old	13.6	24.9	27.6	25.5	21.9	5.9
Guarnerius B (1728)	old	12.9	23.7	25.1	27.9	24.1	10.1
Guarnerius F (1738)	old	11.9	22.3	27.9	25.3	22.1	13.5
Guarnerius average		12.2	24.1	26.3	26.2	23.5	10.0
Old Italian violins; makers probably as indicated							
Maggini	old	15.7	23.7	26.3	23.3	22.5	12.3
P. Guarnerius (Cremona)	old	14.6	19.4	22.8	28.7	27.6	15.7
Stainer	old	14.5	21.3	24.1	27.5	24.5	14.5
A. Guarnerius	old	13.5	24.5	25.7	25.1	20.5	11.9
Stradivarius	old	12.7	22.7	24.9	25.7	23.5	15.1
Guarnerius, J	old	18.1	23.3	24.7	24.1	22.9	11.3
Gagliano	old	18.1	27.1	28.3	22.5	14.7	5.7
Guadagnini	old	15.1	23.5	24.1	25.1	21.1	14.1
Pressenda	old	12.5	22.3	24.3	27.1	22.1	16.7
These old violins, average		15.0	23.1	25.0	25.5	22.2	13.0

Violins	Method	I	II	III	IV	V	VI
New violins							
J. A. Gould (1382)	old	13.5	22.9	27.3	25.7	25.5	9.7
W. S. Goss (1912)	old	10.1	24.1	27.7	24.3	21.5	15.1
Sangster's No. 30	old	10.3	26.1	23.5	27.7	23.7	11.3
	new	13.4	22.1	24.6	26.2	24.6	13.3
Sangster's No. 33	old	12.9	26.1	26.1	25.3	21.3	9.0
Sangster's No. 34	new	13.0	23.2	21.0	27.4	27.1	13.9
Sangster's No. 35	new	12.4	23.5	23.8	25.6	28.9	11.0
Sangster's No. 37	new	11.7	24.9	23.3	25.8	26.2	11.9
Sangster's No. 38	new	12.6	22.5	24.4	26.6	23.2	13.8
Moennig, copy of Strad B	old	12.1	24.0	24.3	24.7	28.3	15.3
	new	11.3	21.3	27.8	26.2	23.2	13.4
Moennig, copy of Strad J	old	11.1	24.5	26.3	23.1	21.9	16.9
Moennig, copy of a Strad	old	14.5	21.3	22.6	23.6	28.8	17.8
Moennig, copy of a Guadagnini	old	15.2	20.8	26.4	22.7	22.3	17.9
Yurkevitch	new	12.7	24.4	27.1	25.3	22.0	8.6
B. F. Phillips	new	13.1	23.5	25 2	26.8	22.9	10.0
Koch	old	15.3	21.9	25.9	26.7	18.9	14.1
	new	11.0	23.8	23.2	26.2	24.9	11.5
Stanley	old	12.5	20.9	26.5	28.1	20.7	15.5
	new	13.5	23.6	23.1	26.9	25.9	10.7
Moglie (1930)	new	11.5	23.3	29.3	26.2	22.6	6.9
Averages of good new violins		12.7	23.3	25.1	25.8	24.0	12.7
Two bad violins							
Violin X ($5.00)	new	13.9	18.5	29.5	23.4	22.6	17.6
Violin Y	new	2.6	20.5	20.9	42.2	17.0	16.4

FIG. 5

RANGE: I 196-349 Hz
II 349-784 Hz
III 784-1568 Hz
IV 1568-3136 Hz
V 3136-4186 Hz
VI 4186-6272 Hz

④

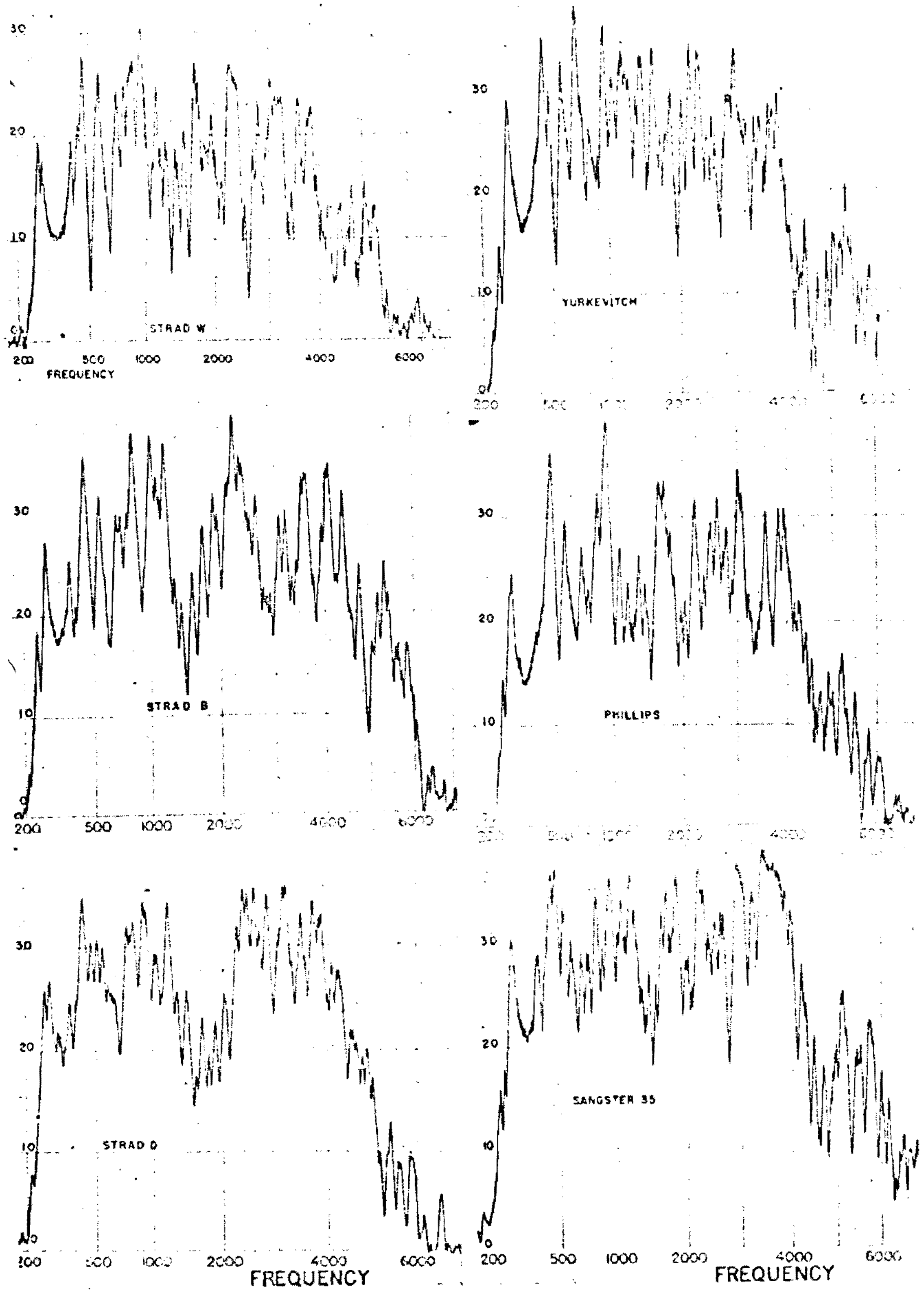


FIG. 1. Response curves by new method of three Strads, and three new violins which led in the Heifetz test.

FIG 6

conducting these tests, he used as standards the instruments persons of Sascha Jacobson and Jascha Heifetz. He used, as a definition, the generally accepted "good" violins as a standard of reference, and recorded loudness curves of various tones. He found certain features of the tones of Stradivaris, Amatis, and Guarneris, that were common to them all. Amongst these were, the strong carrying power of the instrument as a whole, with a fine, clear tone. Analysis of the tones showed that each violin had a variety of resonances, and that these resonances were spaced in such a manner as to reinforce certain parts of the spectrum. Subsequent research has centered on determining exactly which resonances, how they are spaced in the spectrum, and what variations in wood, wood thickness, varnish, etc., will give rise to the desired results.

GOOD VIOLIN TONE CHARACTERISTICS

Various researchers have come up with slightly different criterion for what constitutes the ideal violin tone. Part of the disagreement comes from the use of different violinists in the subjective testing. Chamber musicians will prefer a softer, more mellow tone than the soloist with a symphony, who usually likes a somwhat more piercing tone. In any event, there is still fairly universal agreement on some of the aspects of violin tone. These include;

1) large amplitudes (of sound intensity) at the lower frequencies
2) low amplitudes at high frequencies (generally above about 3,000 Hz)
3) small amplitudes in the region of 1,500 Hz

4) accentuation of the spectrum in the region around 2,000-3,000 Hz

The fact that the lower frequencies are preferred is explained qualitatively by saying that this lends greater carrying power to the instrument. The tone is fuller and more pleasing. The higher frequencies are more piercing and generally less desirable, except in the case of solo instruments for symphony work, where it is desired to have a tone that will cut through. Even then, the difference between good chamber music instruments and good solo instruments with respect to actual sound intensity is relatively small. On the other hand, the most common problem with very cheap violins, and the reason that they sound so bad is the large amplitude of their spectrum in the higher frequencies. This imparts a shrillness in the tone that is unbearable to many people.

The lower frequencies then make the tone sound full and pleasing. The particular conditions of the 1,500 Hz region and the 2,000-3,000 Hz region are explained qualitatively. In the former case, the small amplitudes of the 1,500 Hz region does away with the "nasal" sounding quality of the tone, which is generally regarded as undesirable. The accentuation of the 2,000-3,000 Hz region on the other hand is said to lend a desirable "sonorous" quality to the tone. At any rate, this is what violinists have decided that they like, and most of the finer, famous violins have these qualities*.

SEE FIGURES 1-3-4-5-6

SUBHARMONICS, BODY & AIR RESONANCES

The idea of a harmonic series is very familiar to anyone who has worked with acoustics. Generally, given a certain tone f_o, when generated on a real instrument gives rise to multiples nf_o of that tone, where (n) is an integer. The tone f_o is called the "head tone" of the series, and is the lowest member of the series. The subharmonics of a series are given by f_o/n. They occur in the same ratios as the harmonic frequencies, for example as $f_o/2$, $f_o/3$ ..., but the subharmonic series cannot be excited by the head tone as in the case of the harmonic series. Rather, each of the tones corresponding to the subharmonic series must be individually excited for it to exist.

The usefulness of the subharmonic series is as follows. It is known that if one harmonic of a complex tone is strengthened, then the ear will hear this change as an increase in the loudness of the whole tone with no change in the pitch, but with a slight change in the tone quality. This is beneficial to the lower frequencies of the violin which are often somewhat weak, in that the instrument usually contains many subharmonic series of its own. The natural resonances of the wood and the body are all head tones of these subharmonic series, and lend support to the lower tone by strengthening the harmonic of the tone to which the resonance corresponds. Thus by proper use of resonances in the body, the tone can to some extent be tailored to the needs of the particular individual.

At the end of this section are examples of curves made by various researchers using various excitation and detection methods, on violins of varying quality. The following sections will point out some of the highlights of these curves.

Air Resonance

Every closed volume of air pocesses a certain resonant frequency, somewhat akin to the idea of a Helmholtz resonator. The enclosed air of the violin body likewise corresponds to such a frequency. In this case, the air peak, as it it called, is shown* as a wide, strong amplitude on the amplitude curve. This peak gives rise to a very strong fundamental, which because of its width (because of a high damping factor) enriches and reinforces the tone of the violin in a region several semitones wide surrounding it. According to the results of C.M. Hutchins,[11] this air peak should be centered around D 294 Hz on the violin in order to be most beneficial.

Because the air peak is so important, several articles have dealt, at least in part, on the effects of various parameters on the air peak. Basically, two things can be done to change the air peak; changing the size of the enclosed volume, and varying the size of the f-hole openings. In the first case, it in known that the frequency is inversly proportional to the square root of the volume, or in the usually working sense, inversly proportional to the depth of the instrument, as this is the parameter that is usually varied.

The effects of the f-holes are a little more interesting. A change in area of the f-hole results in a frequency change proportional to the fourth root of the area of the opening. Therefore, a 25% increase in the area of the opening will result in a frequency rise of only about a semitone. This correction factor limits its usefulness "fine tuning" only.

Saunders[18] conducted tests to measure the effects of the inner air alone on the tone of the violin. He plugged the

*SEE FIGURES 1-3-4-5-6

f-holes with very soft cotton (as to stop the vibrations in the hole, yet not interfer with the free vibration of the top) and took amplitude curves of the instrument. He then compared these curves with those of the instrument with the holes open. The plugging of the holes resulted in the following observations;

1) the intensity of the open G string decreased from 90.0 dB (open) to 86.8 dB (closed), a loss of 3.2 dB
2) the intensity of the D string decreased from 90.0 dB to 88.8 dB, a loss of 1.2 dB
3) the test on the other two open strings showed; the A string, 89.1 (open)/ 89.0 (closed) the E string, 85.5 (open)/ 90.1 (closed), indicating no change within the limits of accuracy of the experiment.

The air peak enhances the tones in about a seven semitone region around itself. This corresponds to roughly B-flat to E on the G string, and to some of the lowest tones on the D string. In addition, the tests showed that there was no measurable vibration air in the region of the f-hole except near the frequencies of the air peak. This was tested by various methods including observing the motions of a small feather suspended in the f-hole through a microscope, while the violin was excited.

Body Resonances

The violin body contains only one main air resonance, corresponding to its enclosed volume. On the other hand, there are usually many body resonances. Hopping, Hutchins and Saunders[9] found that many good instruments, including Heifetz's Guarnerius, can have fourty or more peaks in their intensity curves. These peaks are the results of various combinations on the resonances of the individual plates of the violin, the resoncances of the body itself, and of the preferred tones of the wood. These peaks in the curves are also often referred to a "wood resonances".

The spacing of these peaks adds to the overall power of the instrument by supplying the head tones for many sub-harmonic series.

There are many factors that are related to the placement of these peaks, including the type of wood, its shape, thickness, balance and internal friction. As an example, the less friction, the more readily the sound will travel through the wood, and the greater the effect of resonance will have between the head tone and a harmonic of lower tone. C.M. Hutchins[11] claims that the main body resonance should fall at about A 440 Hz, which corresponds to the open A string, in that this is what she has found to be a common feature of the many Amatis, Stradivaris, and Guarneris that she has tested.

A considerable amount of time and effort is put into the adjustment of the various body or wood resonances. The thinning of the top and back plates, and the technique of tap tones play a vital part in making a good violin, perhaps more imprtant than that of the air peak. This will be discussed in the following sections.

TAP TONES & PLATE TUNING

The number and placement of the body or wood resonances in the plates of the violin are vital to the final sound of the instrument. Generally the condition of the top plate is the most important, with small variations in the thickness at different places affecting the final tone considerably.

The use of tap tones is common to all violin makers and even a few guitar makers. Basically what is involved is the determination of the natural frequency of vibration of the free plates before they are attached to the body of the instrument. The violin maker holds the plate (front or back) at thumb and forefinger distance from one end of the plate near the midline. The wood is then given a sharp tap with the knuckles at various points. This results in various tones of definite pitch being produced. The clearest tones are usually produced when the plate is tapped in the center.

Savart[6] concluded from his tests that "... a top of spruce and a back of maple tuned alike produced an instrument with a bad weak tone". Rather, it has been found[9] that many good violins are tuned with the top having a main tone between C-sharp$_3$ and D$_3$, and the back tuned to D$_3$ or D-sharp$_3$, the top always one tone or one semitone lower in pitch than the back. Other violin makers however, claim good results with the plates tuned alike. While neither of these results has universal approval, the literature taht I came in contact with seemed to generally favor the un-alike tuning pattern.

Hopping, Hutchins and Saunders[9] use a particular pattern of tuning the plates, not by just the main resonance, but some of the smaller resonances also. They have found that they get the best results by alternating the the main peaks of the top and back. They have used two different schemes;

$$T_4\ B_3\ T_3\ B_2\ T_2\ B_1\ T_1 \qquad \text{and} \qquad T_4\ T_3\ B_3\ T_2\ B_2\ T_1\ B_1$$

where T_4 and B_4 correspond respectively to the highest tone of top and back. In the first scheme, T_1 is higher than B_1 and in the second, B_1 is higher than T_1.

In good sounding instruments, these tones alternate at intervals of a whole tone or a semitone. This arrangement promotes a powerful, even quality on all of the strings. On the other hand, if the spacing of the intervals is closer together or farther apart, the tone suffers, giving rise to a tone that is either harsh and gritty, or too nasal.

The different tuning schemes result in slightly different tonal qualities. Using the first scheme, the instrument is of the type preferred for orchestra work, with a somewhat more piercing quality and more power. The second arrangement results in an instrument more suited for chamber music. It pocesses "rich low tones and ease of playing". To achieve this tuning however requires extensive thinning of the plates, and therefore weakens the instrument structurally. This should be considered if the violin is to withstand hard use. As a final note, it should be added that the first type of scheming can be converted to the second by thinning of ~~hte~~ the plates.*

The average tones of the plates increase when glued to the instrument, because of the stiffening effects at the edges. The change usually amount to about a 7 semitone rise of the body tone above the average of the free plates.

SEE FIGURE 7

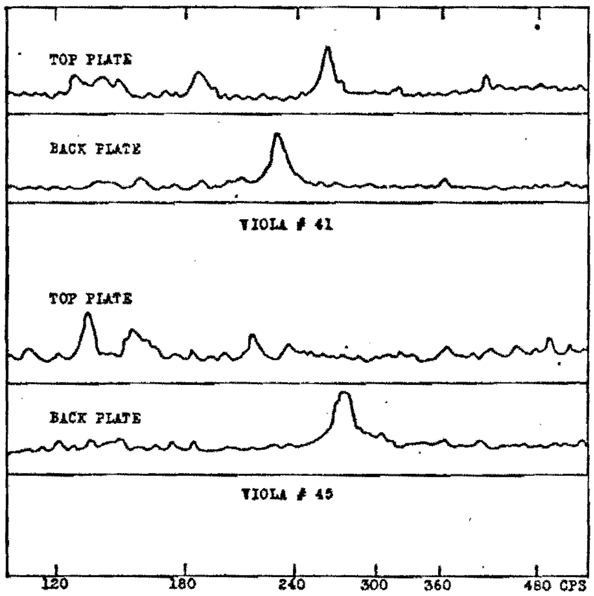


Fig. 5. Tracings of photostrips of the free top and back plates of Viola No. 41 which is preferred for orchestra and solo performance; and Viola No. 45 selected for its "rich low tones and ease of playing."

FIG 7

EFFECTS OF WOOD THICKNESS ON THE RESONANCE CURVE

By varying the thickness overall and in certain places on a plate, the plate may be tuned to the proper pitch by the violin maker. The tuning can result in an instrument with a tone on either side of the desired, either too shrill or too bland. The optimum is controlled by experience of how much material to take away and from where. The plate is not uniformly thick throughout, and small changes in certain areas can greatly af- the final tone.

For instance, the 500 Hz region of the sound curve can be enhanced by the gradual reduction of the thickness of the wood on the left middle bow of the back, according to Meinel.[13] Other changes in certain parts of the plates do similar things. Perhaps the most promising results however, were from Saunder's[18] work with the so-called "wolf tones".

Many violins, especially cheaper ones, have one body tone that is considerably more pronounced than any of the others. This "wolf tone" is a highly undesirable feature, as the violin will "howl" uncontrollably when this resonance is hit. Therefore, Saunders tried to find a way of removing this feature from the instrument. The first attempts centered about stiffening the top with pieces of wood glued to the surface. This resulted in the removal of the wolf tone, but at the expense of the tone in the rest of the spectrum.

It was then figured that since the wolf tone is after-all merely an unstable vibration, that perhaps by loosening the edges, the wolf tone could be removed. Thus a small ditch, about 1 mm wide and about 1 mm deep was cut around the edge of a viola, just inside the line where the top was glued to the body. The area underneath the finger board and the tail piece however, was left intact in order that the top could continue to withstand the tension of the strings. This alterration cured the viola of its wolf tone, by changing the spacing, and increasing the intensity of the body peaks. The process was later applied to a number of other instruments with very good results. One $5.00 violin used by Saunders as as a "standard of badness" for many of his previous tests was treated to such an operation and later was chosen as comparing to the old Italian violins in tone quality in a blindfold test. The top is then found to be very sensitive to changes in the top thickness. (It should be noted that the edges of the old Italian violins were themselves very thin averaging about 2mm. As modern orchestras tune to a somewhat higher pitch than in Stradivari's time, violins have been being built with somewhat thicker edges to withstand the added stress. But this trend has been reversing itself in recent years as the results the thin edge has become more well known.)

It should be added in closing this section that there are some miscellaneous ways to vary the peaks. These are generally regarded as being unimportant, but they include such practices as loading the bridge with a heavy mute or otherwise changing its mass or shape in order to lower the position of the peaks.

WOOD

The choice of the proper material for constructing any musical instrument is, of course, very important, in that much of the tonal character will be dependent upon the not only the type of material, but its age, grain structure, and other characteristics. The violin especially is sensitive to the choice of wood used in its construction. The vibration of the top plate in particular, and to a lesser extent the back and sides, gives rise to the many resonances that give the instrument its tone. In addition, tonal analysis of good instruments show that the amplitudes at high frequencies should be small while the amplitudes should be high. A combination of working with the wood's shape and thickness, and the intrinsic properties of the wood itself contribute to the violin's aquiring these properties.

According to Davidson,[6] "deal" or spruce should be used for the top of a violin. He states that its low density and elasticity are such that it is ideally suited for this purpose. He states that the resistance to flexion of "deal" is the greatest of the common woods suitable for instrument construction. He also argues that the mass to elasticity ratio is favorable. As a final note he adds that while the velocity of sound in "deal", glass and steel are approximately the same, neither of the others pocess the inhomogeneity of velocity in different directions. "Deal" conducts sound at about 15-16.5 times the velocity of sound in the air parallel to the fibers, and about 2-4 times the velocity in air perpendicular to the fibers.

In spite of these observations, the reasons as to why spruce or pine should be used remained unexplored until relatively lately.(In addition, some of Davidson's observations are somewhat suspect in light of research since 1881). In any event, more recent work, the earliest that I could find being Abbott and Purcell's article from 1944[1], starts to look for the particular characteristics of a good wood necessary for violin construction. Basically what has been discovered is that the wood must be light enough to allow for free vibrations of the plates and yet be strong enough to withstand the stresses and tension of the strings. In addition, the damping properties of the wood are important in producing the desired tone.

H. Meinel[14], in his article, sites the results of E. Rohloff (Physik, 117, 1940), who conducted tests on the logarithmic decrement of bending for various wood samples. Rohloff's results showed that the decrement was independent of frequency, from 10 Hz to 10,000 Hz. These results seemed a little strange, so new tests were run. Meinel's tests indicated a definite trend towards damping by the wood at high frequencies.* The graph shows the results of these tests. Spruce, it will be noted, has the least amount of damping at the lower frequencies, and rises the most quickly at the higher frequencies. This is in accordance with the desirable feature of good violin tone; large sound amplitudes in the lower register, and a supression of the higher frequencies. The results of Meinel's tests have been comfirmed by later work, and their relation to the quality of violin tone seems certain.

Meinel's tests and the tests of others, B.A. Yankovskii[23] in particular, show some interesting differences in the response of different woods, and the affects of aging.° Meinel ran tests on maple, elm, oak, ebony, mahogany, and rosewood in addition to spruce. The damping of spruce at high frequecies has already

* SEE FIGURE # 8 ° SEE FIGURE # 9

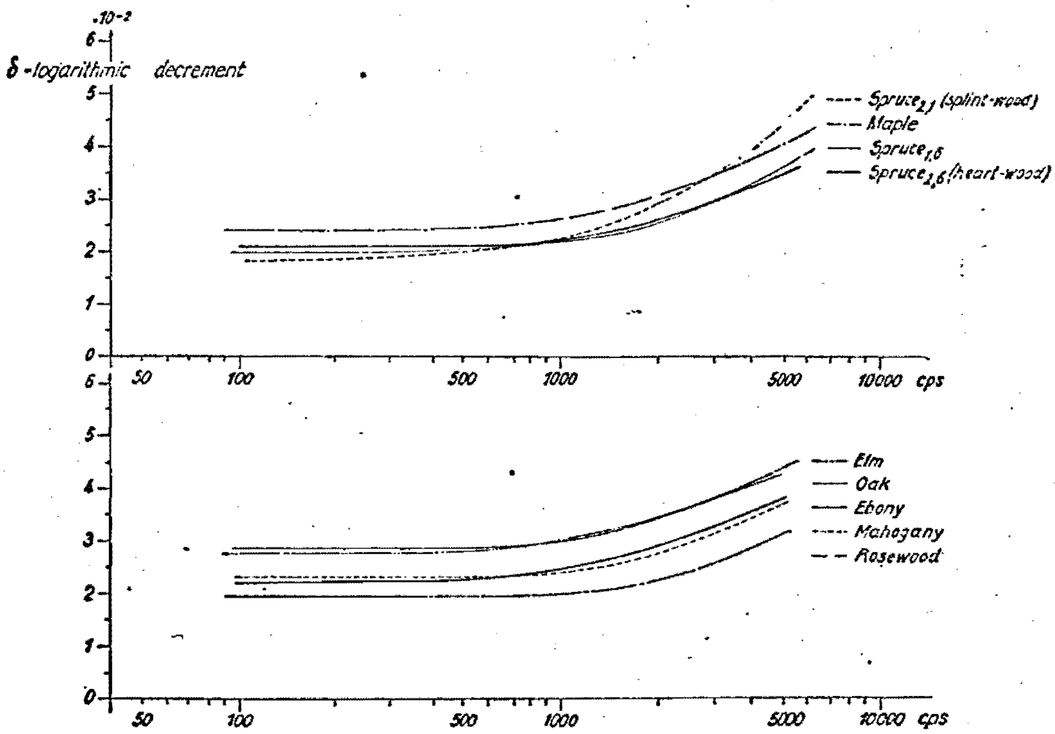


FIG 8

~~10~~. Loga-
...mic decrement of
...rent kinds of
...d. According to
...ent measure-
...nts, the damping
... pine (spruce)
...ibits a greater in-
...se with the fre-
...ncy than does the
...ping of other
...s of wood.

FIG 9

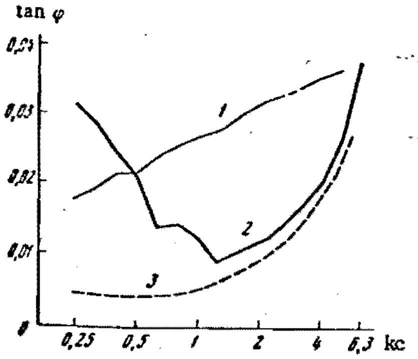


Fig. 1

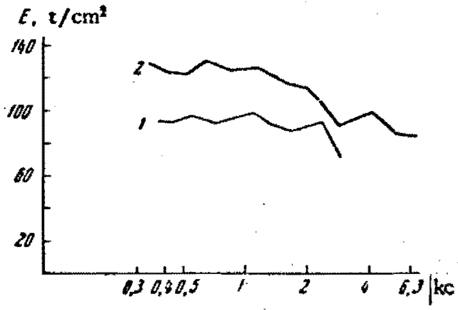


Fig. 2

been noted. It should be observed however,* that the damping curves for mahogany and rosewood show a lesser trend for damp- at higher frequencies. In addition, there is less damping at the lower frequencies. These conditions lend themselves well to the needs of fretted instruments, where the volume is of much importance. Most good guitars will have the entire body (except for the top) made of mahogony or rosewood, with rosewood being the preferred wood for classical guitars.

The damping properties of the spruce samples doubled as the frequecy changed from 100-5,000 Hz. This was about twice that of the other woods. The differences between the damping of the different parts of the wood should also be noted. The sapwood, or splintwood of the samples damped better than the heartwood. Many violin makers take this into account by placing the sapwood at the center (when making a two piece back), and the heartwood at the edge.

Yankovskii has investigated the effects of aging on the properties of wood. It is generally thought that the older a violin, the better it sounds. The tests dealt with three samples, one aged only about six months, one about 170 years old, and finally a third piece approximately 220 years old. As can be seen from the graph,° the curves for the two older samples nearly coincide at high frequencies. This graph is measuring the loss tangent versus the frequency. The loss tangent is a measure of the internal friction, and is given by;

$$\tan = \frac{f_2 - f_1}{f_0\sqrt{3}}$$

where f_0 is the resonance frequency, and f_1 and f_2 are the half peak amplitudes below and above resonance respectively.

* see figure 8 ° see figure 9

The second figure shows the elastic modulus, E versus frequency. The elastic modulus is given by;

$$E = f_0^2 \, (l/b)^3 \; \frac{M-m}{a} \; 0.965 : 10^{-6} \; kg/cm^2$$

where f_o is the resonant frequency in cycles per second, l is the length of the speciman in centimeters, b is the thickness in centimeters, a is the width in centimeters, M is the mass of the specimen in grams, and m is the weight of the vibrator attached to the specimen for its electromagnetic excitation. There is a definite trend greater elasticisty for aged wood at low frequencies than fresh wood, with the curve for the former also showing a marked tendency to decrease in elasticity at higher frequencies.

VARNISH

The relative effects of violin varnish on tone have long been discussed by muscians and craftsmen alike. The power of Antonio Stradivari's varnish is legendary. Many people have thought that there can never be violins made that sound as well as Stradivari's, or those of Guarnerius and Amati, until the long lost secret of their varnishes is rediscovered. The bulk of modern scientific investigation on the effects of varnish tend to minimize it's importance however, although the effects of varnish can be important in extreme cases.

The first real study of the effects of varnish were published by H. Meinel in 1937. He published curves showing the response of violins as affected by certain types of varnish. This report showed that the effects of the varnish were small when compared to other factors in the construction. The largest effect showed in the overall reduction of the amplitudes of the lowest wood resonances by about 2 dB, while the corresponding minima remained about the same.

A later article, published in 1957,[14] showed graphically the effects of certain varnishes on different kinds of woods. These tests, run with samples of wood about 400 mm long, 20 mm wide, and several millimeters thick, spanned the frequency band from 100 to 5,000 Hz. Samples with only half the width of those used in the final test gave essentially the same results, thus allowing the effects of radiation damping to be ignored. At any rate the curves* show that while the wood itself causes some damping of vibration in the samples (with the logarithmic decrement of damping increasing as the frequency increases), the addition of a varnish to the wood makes this effect more

*SEE FIGURE 10

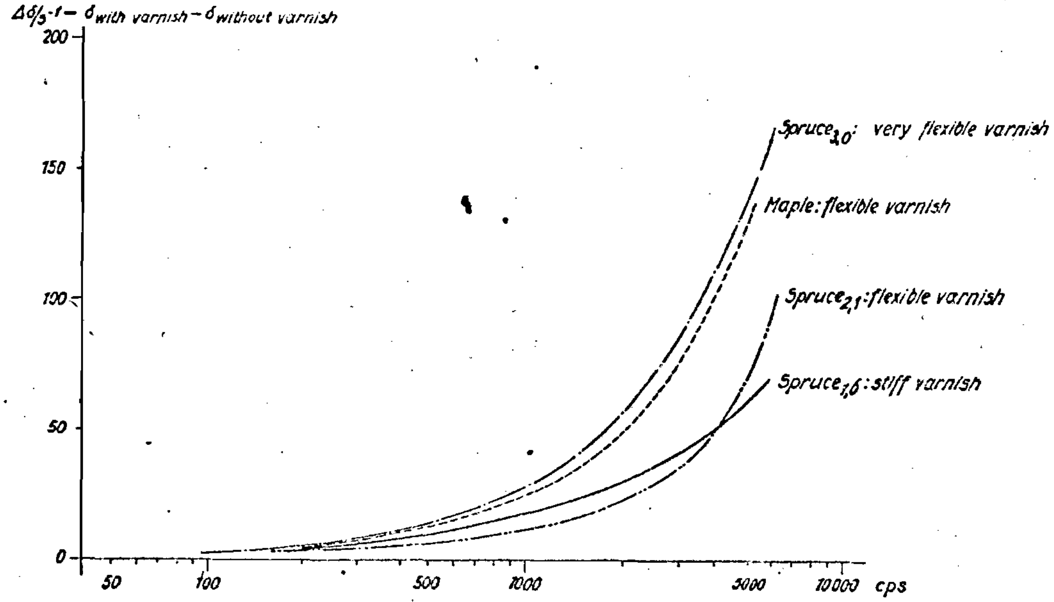
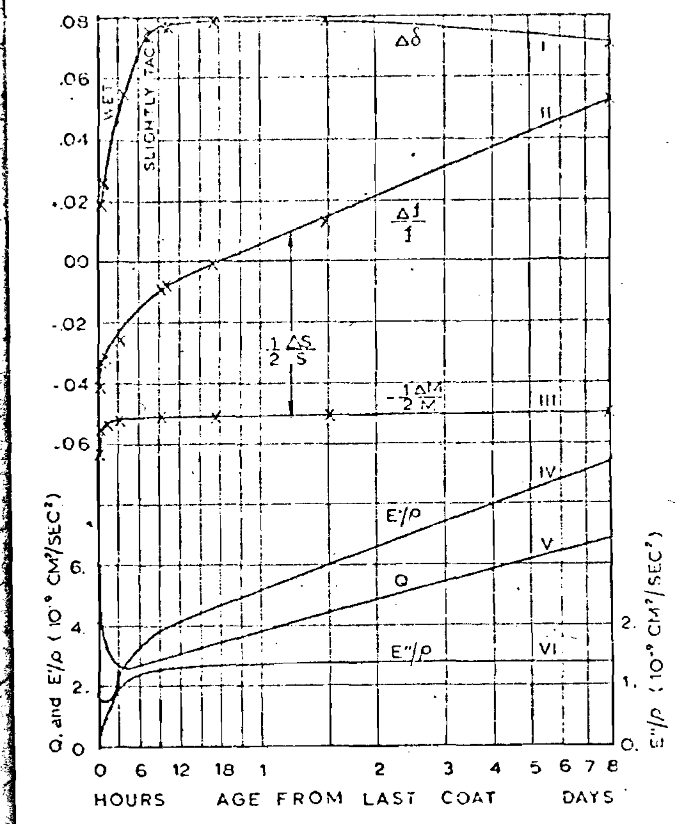


FIG 10

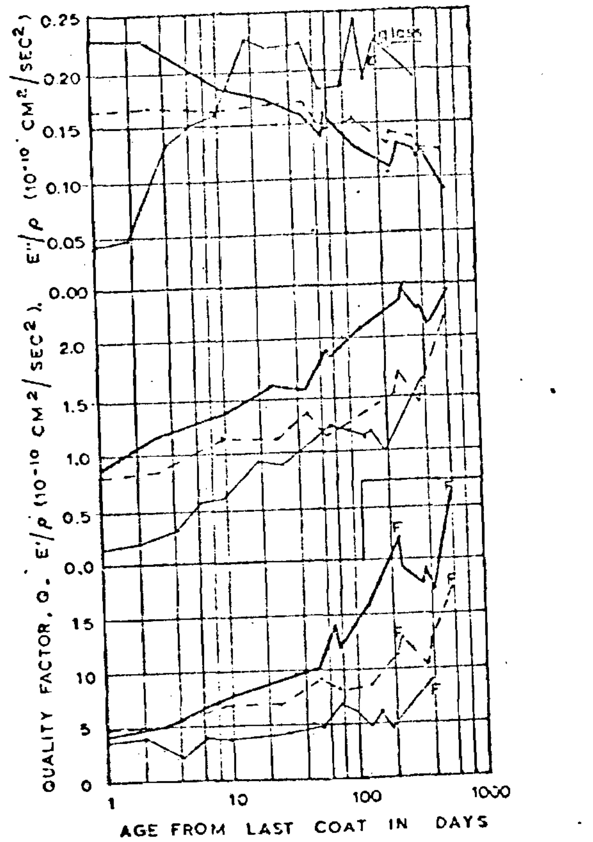
[redacted]9. Change ... the logarithmic ...ement by var- ... A hard varnish ...ases the damp- ... at high frequen- ... less than does a ... varnish.

* This varnish was produced by Dr. Ing. Karl Letters, Köln-Lindenthal, Sielsdorfer Str. 9. During the investigations the soft ...nishes were not quite dry.



[redacted]. Early stages in aging of a moderately hard oil varnish: Curves I, II, and III give experimental data for a varnished strip of wood; Curves IV, V, and VI, deduced therefrom, show real and imaginary Young's modulus over density and quality factor Q for varnish alone. Δ indicates changes induced by varnish.

FIG 11



[redacted]2. Aging of three oil varnishes over an extended period: the middle curves are for the real components of Young's modulus over density, the top curves are for imaginary component and the bottom curves for their ratio Q. Heavy lines for a hard varnish, light lines for a soft varnish, broken lines for a tung-oil floor varnish. "F" means February.

FIG 12

pronounced. The trend to decrease the amplitude of the higher frequencies is regarded as good in a violin. Subjectively the instrument is said to "speak" better, with a decrease in roughness, providing a more pleasant sound. The total effect of the varnish, according to all reports,is not major, however.

The article by Schelleng[20] lends support to the ideal of a very thin coat of varnish being the most desirable, if in fact a varnish must be used at all. He states that many violin makers feel that "the best sounding of all violins, the Cremonese, had the thinnest varnish", and that many also feel that a fiddle actually sounds the best when it is "in the white", i.e. before it has been varnished at all. The effects of varnish can be pronounced when poorly applied however, as the difference between a poor violin and a good one can be a difference of only 5 dB averaged over an entire response curve. If a very thick or hard varnish were to be applied to an instrument, the effects on its tone could be disasterous.

The effects of varnish can be divided into three areas; the effect of added mass, the effect of added stiffness, and the effects of internal friction. All of these are important in the final outcome of the violins's tone.

By the application of a coat of varnish, the effects of added mass to the plates is immediately noticable. The effect is far from negligible, in that a layer of varnish .005 inches thick,adds about 7 grams, or about 10% to the weight of the top plate. This can lower the tap tone plate frequencies about 5% or most of the part of a semitone. The effects of added mass are the same across and along the grain.

As time goes on the effects of the added mass decrease a little as the more volatile components evaporate, and the effects of increasing stiffness increase. Eventually the two factors cancel somewhat as far as the tap tones of the plates are concerned, as the added stiffness raises the frequency. There is however, a residual effect that is important, in that the stiffness of wood, say spruce, is different along and across the grain. In spruce, the stiffness across the grain is some fifteen times less than along the grain. Therefore the effects of the added stiffness are much greater across the grain than along it. This can affect the vibrational patterns of the plate very much.

Along with that effect, even more important is the increase in impedance caused by the additional mass and stiffness. The impedance is roughly proportional to the square root of the product of the two factors;

$$\text{impedance} = \left[(\text{stiffness}) \times (\text{mass})\right]^{1/2}$$

Therefore, a mass increase of 10% and a stiffness increase of 14% leads to an impedance increase of about 12%. This corresponds to an overall response drop of 1 dB. This could be very important in a borderline case.

As for the effects of internal friction of the varnish, it is perhaps best to quote Schelleng;[20]

> Paralleling the effect of the real component of Young's modulus in altering the frequency, there is a loss of power because of the imaginary component. This too, (as with the effects of stiffness) is especially important across the grain. Whereas the impedance effect reduces vibrations in the "valleys" as well as the "hills," internal friction produces little change in the lows, or "nulls". However, it can be very important at the peaks of resonance and is usually the most noticable result of varnish.

noting that in all, he concludes that the internal friction effects are the most noticable in the finished product.

Equations Governing the Effects of the Varnish

The results of this section come mainly from Schelleng's[20] article. The test strips of wood were first measured as to frequency and Q. Changes over a time period in frequency and Q were interpreted in terms of real and imaginary Young's modulus of varnish.

Assuming a simple vibrating system, in which both mass and stiffness can be identified, the angular frequency at resonance is $[(\text{stiffness})/(\text{mass})]^{\frac{1}{2}}$, and the incremental change in stiffness caused by a thin coat of varnish is;

$$1) \quad \Delta s/s = M/M + 2\Delta f/f$$

where M is the mass, and (f) is the angular frequency. The mass M is rigorously the effective mass, not the total mass, but in this circumstance, the difference is unimportant.

The real component of Young's modulus for a film of varnish is;

$$2) \quad E'_v = 1/3\, E'_w\, (H/\Delta H)(\Delta s/s)$$

where E' is the real modulus for the wood substrate, H is the thickness of the substrate, and H is the thickness of the varnish. The factor of one-third compensates for penetration of the varnish into the wood, where it contributes less to the added stiffness than that on the surface.

Since the varnish penetration is sometimes a major effect, ΔH cannot readily be determined in some cases. Interpreting H as the thickness that the known weight would have at the normal density ρ_v, equation 2 becomes;

$$3) \quad E'_v/\rho = 1/3\, E'_w (V/\Delta M)(\Delta s/s)$$

where V is the volume of the wood strip. All of the factors in equation 3 are easily measurable, and thus we have a working equation.

The Q of the varnish is calculated by the equation;

$$4) \quad Q = \frac{\pi(\Delta s/s)}{(\delta+\Delta\delta)(1+\Delta s/s) - \delta}$$

where δ is the logarithmic decrement and $\Delta\delta$ the increment induced by the varnish.

Equation 3 gives the real component of modulus over the density. Q is the ratio of the real to imaginary modulus. The imaginary modulus component is thus obtained from equations 3 and 4 by division;

$$5) \quad E''_v/\rho = Q_v^{-1}(E'_v/\rho_v)$$

Conclusions

Graphs of curves from Schelleng's[20] article* sum up the effects fairly well. The first shows the effects of aging over the immediate interval after the varnishing took place. The curves are fairly representative for all types of varnishes. The second° set of curves show the affects of longer aging. In total, the curve for Q shows some of the more important aspects of what is going on. The relatively high Q at time zero is the result of the volatile components of the solvent. This drops off quickly as they evaporate, giving rise to the internal friction effects, which proceed the stiffness effects. As the tackiness of the varnish vanishes, the effects of the internal friction

*SEE FIGURE 11 °SEE FIGURE 12

cease to increase, while the stiffness effects continue to incraese indefinitely. The internal friction effect continues to be the more important effect, however.

The various conclusions concerning violin varnish are best summed up in tabular form, again by Schelleng;[20]

1) The density of varnish is about unity when compared to wood, with the real component, E' for dry varnish films on the order of 1.0 to 2.5 x 10^{10} dyne/cm^2. This is about the same as the modulus for quarter-cut spruce across the grain, but much less than that along the grain.

2) The real component of Young's modulus increases with age, and a decrease, if it occurs, indicates a change in ambient.

3) The imaginary component of Young's modulus rises early in the drying process, levels off, and then starts a slow decline. During this decline, its fractional change is less than the accompaning rise in the real component.

4) The Q's of all of the varnishes studied ranged from 0.12 (logarithmic decrement) to about 25. With fixed ambient conditions, the Q always increased with age, with hard varnishes reaching about 23 after a year and a half, and soft varnishes reaching 8.5 after about a year.

5) A test with a high grade tung-oil floor varnish failed show any reason why it would be any less effective as a good violin varnish as any of the regular varnishes. It is thought that it may be unacceptable in some nonacoustical way.

6) Meinel's work[14] shows that the decrement of damping increases as the frequency increases. The effect is especially noticable in the range above 5000 Hz.

7) While the differences in varnishes can produce noticable effects in the performances of the instruments, they do not affect the sound as much as legend likes believe. In fact, the best sound is usually heard before the instrument is varnished at all.

8) The impedance that the body of a violin offers to forces exerted by the strings approximates that of a simple mechanical resonant circuit of stiffness S and mass M, quality factor Q, and numerical reactance X, about a given resonance. This impedance is given by;

$$(SM)^{\frac{1}{2}}(1/Q + iX)$$

9) An increase in $(SM)^{\frac{1}{2}}$ decreases the motion in the radiating plates roughly uniformly throughout the frequency range, while a decrease in Q superposes an additional reduction at resonance. This increase in impedance leads to undesirable decreases in sound intensity.

10) In addition to the impedance loss, there corresponds a loss due to internal friction, which can be up to three times as great. The internal friction losses are therefore of major importance.

11) The energy loss in the top exceeds that of the back by a factor of almost three, regardless of the type of varnish coat applied. Therefore the greatest benefits may be gained by reducing the varnish coat on the top.

12) It is feasible to reduce the effects of varnish markedly by proper use of the minimum amout needed to protect the wood. If for example the varnish on top were only one third that on the bottom, the losses would be equal in both directions, the total loss would be nearly cut in half, the frequency difference between the plates would essentially be eliminated, and the change in the "velocity ratio" would be reduced. The net result would be a varnished violin sounding nearly like one in the white.

The reader is urged to read and refer to John Schelleng's article in the Journal of the Acoustical Society of America,[20] as it is very complete. Large parts of this section were taken from that article.

EFFECTS OF DIFFERENT EXCITATION METHODS

When considering the response curves of violins, it has generally been assumed that the method of exciting the body to produce sounds has little effect on the response produced. Some investigators use electromagnetic excitation methods, some use mechanical means, while others prefer to have the violin actually played by a violinist. Each of these methods has its advantages and disadvantages. Having a violinist play is of course the most realistic way to proceed, as this aproximates the actual operating conditions best. However, the use of hand bowing leads to variations in in bow speed, pressure, and other factors that prevent accurate reproduction of results. Electromagnetic excitation, (usually by means of a transducer coupled to an audio signal generator), allows for a continuous response curve by merely continuously increasing the exciting frequency. But this method is very dissimilar to normal excitation. Mechanical excitation, by means of a belt or disk of rosined material, comes closer to approximating hand bowing, while producing more stable results. Its drawbacks have been the somewhat cumbersome equipment needed, and until Bradley and Stewart's[4] work, the lack of a method with which to obtain a continuous curve. This particular problem has been solved be the use of a fingering device consisting of small rubber roller, which continuously shortens the string, while approximating natural fingering.

Since various experimental groups use different methods of excitation, it is worth while to see if any important differences arise because of this. The most complete investigation into this subject was conducted by Bradley and Stewart.[4] They used each of the three different tchniques and compared the results, noting that there are some very definite differences arising in the response curves depending on the type of excitation.*

The hand bowing tests involved recording the sound pressure level (SPL) of tones bowed at semi-tone intervals. The detection equipment used was valid throughout a band of 20 to 18,000 Hz. The results were tabulated in a "loudness curve" that plotted the SPL in decibles versus the frequency. In order to try to make the results as constant as possible several proceedures were introduced, including using a marked bow and strobe equipment to allow the violinist to keep the bow speed as constant as possible, and making sure that the bowing was always done the same distance from the bridge (1¼ inches from the bridge in both the hand and machine bowed tests). The need for controls on the bow pressure were avoided when tests showed that the SPL varied only slightly with a change in pressure, while the bow speed affected the SPL in a roughly linear way within the range of 20-120 cm/sec.

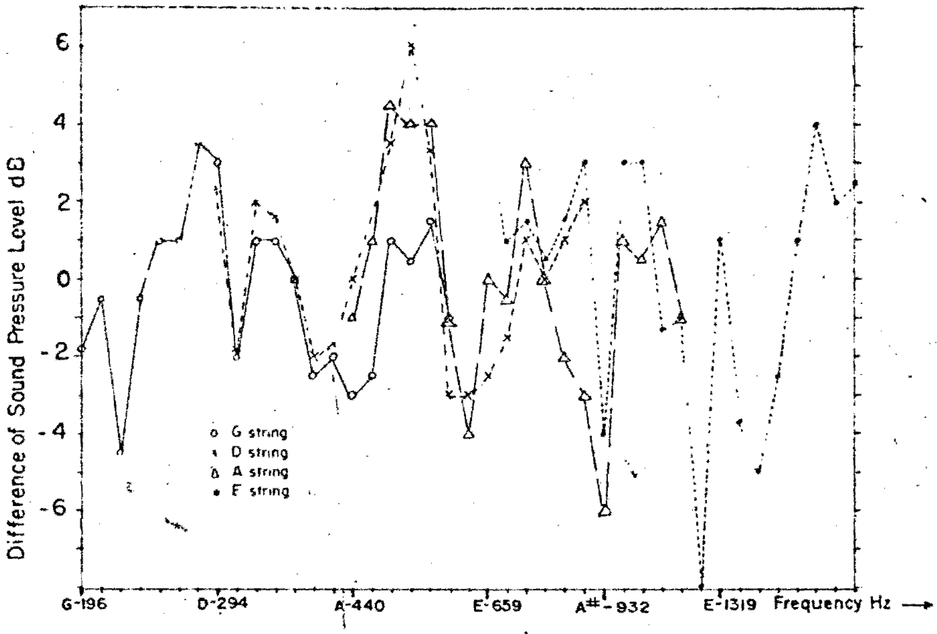
In the case of mechanical bowing, a continuous cloth belt with synthetic bow hair sewn on, supported by two pulleys was driven by a constant speed motor, (which of course was acoustically insulated from the test), at a speed of 30 cm/sec. A continuous curve was obtained by means of the rubber roller fingering device. The results were accuarate for a single frequency to plus or minus 0.5 dB.

In order to test electomagnetic excitation, an electromagnetic activator was attached directly to the bridge. This in turn was driven by a sine wave generator that continuously varied its pur-tone frequency, The device was attached to the G-string side of the bridge.

The final results of the tests brought to light some

SEE FIGURE 13

VIOLIN RESPONSE CURVES BY BOWING AND DRIVER



Differences between and machine-bowed response sound levels.

FIG. 13

very interesting things about the differences between the various types of excitation, and some incidental information concernig the acoustic effects of the violinist himself.

The most important result to come out was the close correlation between hand and machine bowed response curves. In this test of 80 hand bowed tones, almost 80% of the response levels were within plus or minus 3 dB of the machine bowed values. This result makes clear that machine bowing is a very good approximation to hand bowing. With the difficulties inherent with hand bowing, this legitimizes the use of a machine in its place.

On the other hand, the electromagnetic technique did not fare as well when compared to either of the other methods. The air and wood tones compared fairly well, but some peaks, for example a peak around A# 932 Hz in the bowed response curves, was shifted about a semitone lower, and made somewhat broader with the electromagnetic technique. The apparent reason was that the velocity of the bridge increases with frequency with the electromagnetic tchnique, while not with the mchanical techniques. The two systems thus seem to be dissimilar with regards to energy imput, and therefore yield different results. This stems from the fact that the bridge varies in the point about which it vibrates as the frequency varies. This corresponds to a slightly different mechanical system for each frequency, which causes the bow to provide a slightly different force to the bridge. The elcetromagnetic technique however, provides a constant force, thus varying from the natural processes.

MISCELLANY

G. Meinel[13] has done some research into the area of nodal patterns on the top plate of the violin. While he found that the pattern on the top can consist of 20 or more individual nodal lines superimposed, he also discovered that the differences between the nodal patterns of good and bad violins are not significant enough to produce any useful results. However the features of the nodal patterns are interesting enough in their own right. Every component of sound in the complex tone generates its own set of nodal lines. These superimpose to form the net pattern.* It was also found that clear nodal patterns appeared only when the amplitudes of vibration of the wood were great, that is at the wood resonances corresponding to the fundamental body frequency.

In other research, Hoppin, Hutchins and Saunders[9] ran tests to see the effect of varying the density of the fluid in the interior of the violin. In this case they substituted carbon dioxide for air inside the instrument. The net result was a lowering of the "air" tone by 2-3 semitones while the body tone stayed the same.

As one final item, we will consider the effects of the sound post, and its purpose for being in the violin. This work was done many years ago by Savart and by Davidson[6]. Without the sound post intact and in place in the interior of the violin, slightly behind the right (treble) side of the bridge, the tone is very weak, and low in intensity. The sound post does more than conduct sound to the back of the instrument from the front. Rather, its primary function is to render the vibrations of the

* SEE FIGURE 14

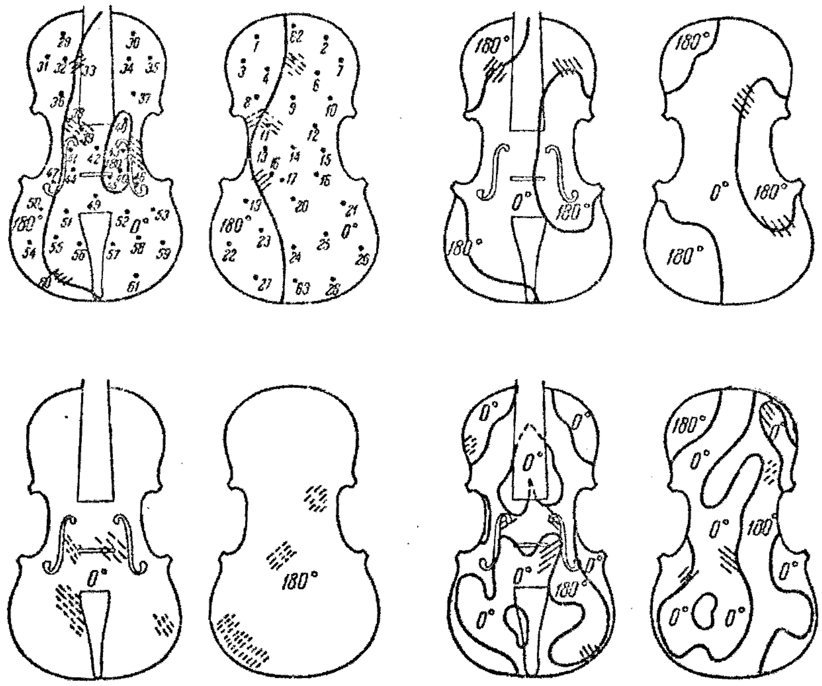


Fig. 1.

At the lowest frequency of the violin (G) one nodal line is seen to traverse the sounding board, extending to the base of the body. The next highest, clearly pronounced nodal-line pattern is obtained for the notes from C_4 to $C\#_4$, i.e., approximately at 280 cps, at the fundamental frequency of the inner air volume of the violin. In this connection, the form of the oscillations is not only determined by the superposition of flexural waves traveling counter to one another over the body of the violin, but by the mutual connection of the body oscillations and the air volume. Here the nodal line passes through the belly. As the pitch is raised, after this form of oscillations there appear increasingly more complex nodal lines, in correspondence with the theory of oscillating plates

G. MEINEL

SOVIET PHYSICS - ACOUSTICS

OCT - DEC. 1960

FIG 14

two plates normal, and to set up a node at the right foot of the bridge. The first result is tested by bowing the instrument both parallel and normal to the belly. In the first case, (in both cases the sound post has been removed) the sound is very weak, but in the second, the tone is almost the same as with the sound post back in. To test the second theory, Savart tried various methods of creating nodes at the right foot of the bridge, and found in all cases the tone increased greatly.

SOUND PROPAGATION TO THE ENVIRONMENT

As the sound of a violin must carry away from the musician and travel a considerable distance to the audience, some consideration should be given to the features of the sound's propagation. Meinel[13] ran two very simple and stright-foward tests that show what happens when the sound does leave the instrument.

The first of these experiments was to determine the directional properties of propagation. Taking readings at various frequencies*, he came up with figures that showed that the sound tends to direct itself more and more as the frequency increases. The lower frequencies tend to radiate more or less uniformly in all directions. These features do not vary with respect to the quality of the instrument, but rather seem to be intrinsic to the violin in general.

In addition, it was found that as the frequency increases, the back of the violin radiates less and less. Therefore, the area to which the top is projecting will recieve a larger intensity of sound. In normal playing position this means that the audience should be to the right of the violinist, or on the left hand side of the conductor. The first violinists in the normal seating of an orchestra are usually seated this way, and in addition positioned such that the audience is also at the optimum 315^{o} to recieve the directed high frequency radiation.

In addition to the directive properties of sound emission, Meinel studied the relation of decrease in intensity as a function of distance°. He found that to an excellent approximation, the intensity follows a 1/r relation, independent of frequency, and that this held in spite of the selective directional radi-

* SEE FIGURE 15   ° SEE FIGURE 16

(37)

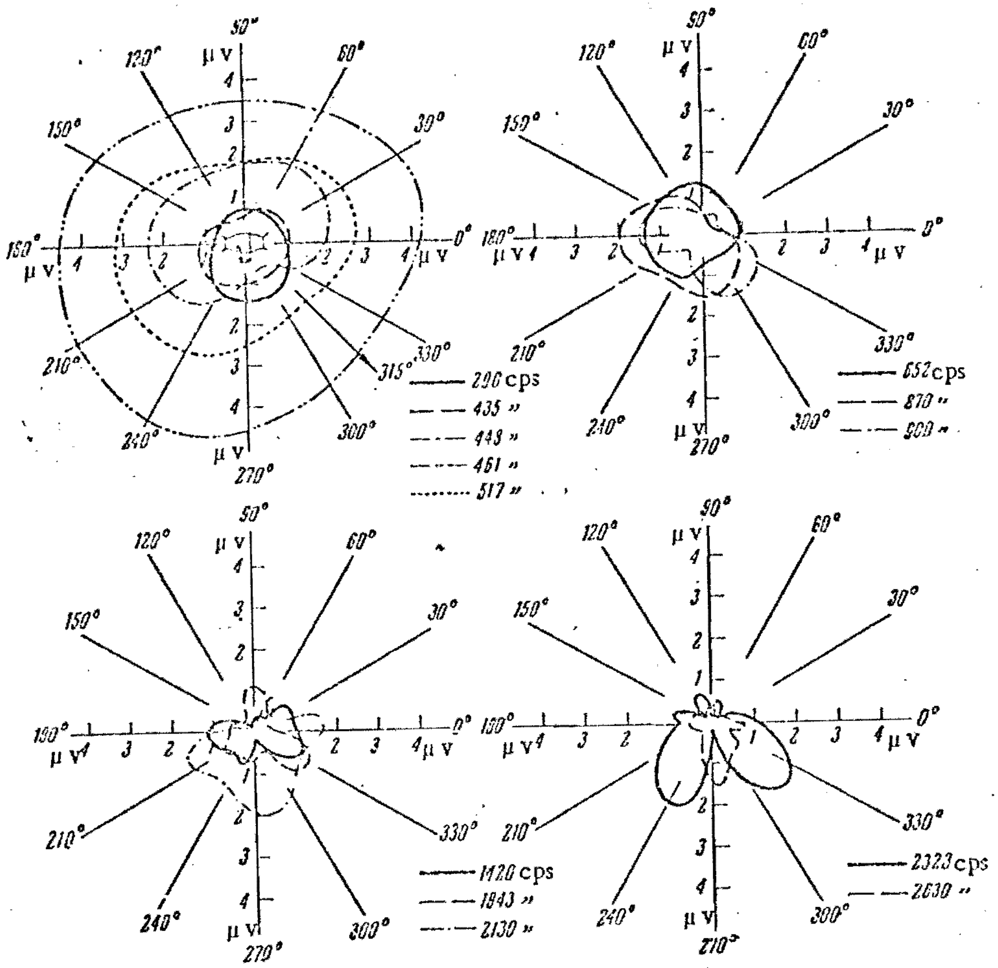


Fig. 2.

FIG 15

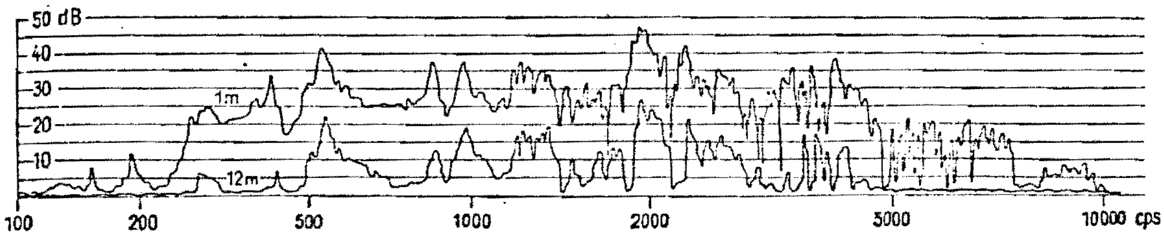


FIG. 5. Violin response curves at distances of 1 and 12 meters. Note that the difference is very near to 21.5 db as required by the inverse-distance law.

FIG 16

ean ... e or o
r the mean curv

— average response curve of very good old Italian violins (ideal curve for violins)

} response curves of bad violins

ation at high frequencies.

This experiment was conducted under conditions where there was very little absorption or echoing involved. In a concert hall however, the highs will be more readily absorbed than the lower frequencies. Therefore it is well that the power of the violin is centered in the middle and low range, for this lends itself to strong carrying power. If the majority of energy were being put into the higher tones, such as with a cheaper violin, the sound would be rapidly absorbed by the surroundings, and the tone would appear very weak. This is one more factor in favor of resonances in the lower range, with damping in the wood and varnish for the higher frequencies.

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 35, NUMBER 3 MARCH 1963

The Violin as a Circuit

JOHN C. SCHELLENG

310 Bendermere Avenue, Asbury Park, New Jersey

(Received 6 August 1962)

The paper applies elementary circuit ideas to bowed-string instruments and their component parts. Parameters are defined and calculations based on simple circuit diagrams for the main resonance and the air resonance; curves describe their combined performance. The relative importance of circuit resistances—wood loss, radiation, and wall-surface loss—is discussed. Wall-surface loss is an important component of air decrement. No material improvement is to be expected from change in decrement or enclosure volume.

A theory for the wolfnote is given in terms of the beating of two equally forced oscillations, together with a criterion for its occurrence and a method for its elimination.

The paper analyzes principles of dimensional scaling between members of the violin family and shows why the cello and viola are more susceptible to wolftone than the violin.

A study of impedance requirements in wood shows that flexural similarity depends on the parameter c/ρ (compressional velocity over density); high values are in general favorable in the top plate. In the violin, cross-grain losses probably exceed those along the grain.

INTRODUCTION

THOUGH the use of circuit concepts is a standard practice in acoustics, in the specific field of the bowed-string instrument they have hardly been emphasized to the degree which their usefulness justifies. The violin family presents many unsolvable problems; its shape and the peculiarities of its materials were certainly not selected with regard to convenience in analysis. This, however, only emphasizes the need for understanding the simplicities that do exist and may even condone a certain amount of oversimplification. It is, therefore, with no thought of novelty that this paper applies elementary circuit ideas to bowed-string instruments, but rather with the belief that something can be gained through representation by circuit concepts and diagrams even though some of the results are only roughly quantitative. These relations lead naturally to such related topics as the relations between different instruments of the family from point of view of dimensional scaling and the physical requirements of the wood.

LIST OF SYMBOLS

A	equivalent piston area
E	Young's modulus
F	force
H	thickness
K	characteristic impedance of string, $(T\mu)^{\frac{1}{2}}$
L	maximum safe load
M	mass
P_0	barometric pressure, 10^6 dyn/cm^2
Q	quality factor of a resonance, π/δ
R	resistance
S	stiffness
$\bar{S}$	area of surface of cavity
T	string tension
U	volume velocity
V	volume of enclosure
W	potential energy per unit area
Z	impedance (Appendix II)
a,b	subscripts for air, body; dimensions of a rectangular plate
c	speed of compressional waves; in air, $c \doteq 3.45\times10^4$ cm/sec
d	diameter of port
f	frequency; subscript denotes resonance
i	$(-1)^{\frac{1}{2}}$
l	length of string or plate
l_1	length of string from bow to bridge
l_2	$l-l_1$
l_{b_r}	length of string having frequency that of body resonance
m	mass loading on bridge
r	subscript for radiation; also radius of curvature of a plate

s	amplitude of vibration
t	time
u	particle velocity
Ω	$f/f_b=\omega/\omega_b$; $\Omega_s=f_s/f_b$
Δ	frequency of separation of fundamental pair
α	(Appendix II)
α_w	acoustical absorption coefficient
γ	ratio specific heats
δ	log decrement to base e, $=\pi/Q=2.30\delta_{10}$
ϵ	low-note scaling ratio (Sec. VII)
η	resonance scaling ratio (Sec. VII)
λ	air wavelength
λ_s	wavelength on string
μ	mass of string per cm
ρ	density; for air $\rho_0=1.2\times10^{-3}$ gm/cc
σ	length (breadth) scaling ratio (Sec. VII)
ω	$2\pi f$; a subscript denotes resonance

I. GENERAL CHARACTERISTICS

In contrast with most of the wide-band radiation systems of today, such as the horns of acoustics and microwaves and "hi-fi" loudspeaking systems, the 17th-century creators of the violin of necessity accomplished their "broadbanding" by distributing through its wide frequency range many relatively narrow resonances, rather than using one or two bands of nearly aperiodic response. The frequencies desired extend from about 200 to 6000 cps, a span of five octaves. In the upper octaves the wood provides body resonances in number sufficient to give a quasiuniformity of response. In the lower octaves this series comes to an end, and the lowest or next-to-lowest resonance—in the vicinity of 460 cps—is commonly called the "main body resonance" because of its pronounced effects. Without reinforcement below this point, response would fall off at a rate of 12 dB per octave or more. Air resonance similar to that in loudspeakers is employed to sustain the volume for the better part of another octave, that is, resonance of the air chamber breathing through the f holes.

It is common knowledge that even a fine violin has strong and weak regions in its frequency range, but the effect is by no means as extreme as the measured characteristics suggest. F. A. Saunders and co-workers[1] point out that the subjective feeling of uniform strength, which a good violin evokes, depends markedly on the well-known effect in which the ear credits the fundamental with an increase of volume actually brought about by a strengthened harmonic. This subjective reduction of depressions occurring in an objective response curve contributes not only to uniformity of loudness, but gives subtle and interesting differences in tone color.

In Fig. 1(a) is shown a violin with names of various parts. Figure 1(b) represents its circuits in terms of electrical symbols in accord with standard conventions of acoustics for the direct or impedance type of analogy. Letters on the circuit correspond to those in Fig. 1(a).

II. THE CIRCUIT IN BRIEF

The circuit begins at point B where the bow rubs across the string giving rise to a negative resistance. In the simplest Helmholtz mode[2,3] for the bowed string, the string at every moment comprises two straight sections either side of a discontinuity which shuttles from end to end of the string around a narrow lenticular path. Ideally the string clings to the bow except for brief recovery periods in which short negative pulses occur. From the circuit point of view, the important result is that the bow-string contact B is a constant-velocity generator. The weight applied to the string by the hand provides a condition necessary for vibration, but is unimportant in its effect on amplitude and velocity.

It is inherent in the Helmholtz concept that capture and release of the string are timed by the shuttling discontinuity that provides the trigger by which the pulse is regenerated. Since sharpness of discontinuity depends on properly phased harmonics more than fundamental, it is necessary to bear them in mind in any question concerning frequency produced. One naturally expects the occurrence of a frequency for which reactance seen by the bow at the fundamental is zero; if this were so, however, the frequency near the main resonance (to be discussed) would depart intolerably from the natural frequency of the string. Thanks to harmonics, whose impedance is independent of the main resonance, this effect is small.

The motion of the contact is in series with the two parts into which it divides the string. Except for high harmonics, Sec. AB is essentially positive mechanical

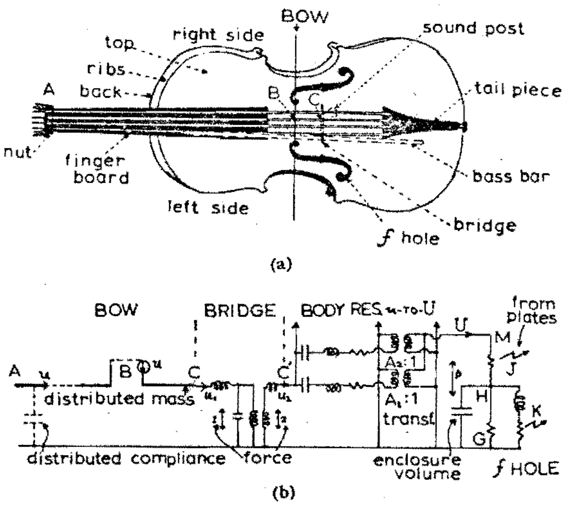


FIG. 1. Violin and an equivalent circuit. A, B, C, and K appear on both diagrams.

[1] C. M. Hutchins, A. S. Hopping, and F. A. Saunders, J. Acoust. Soc. Am. 32, 1443–1449 (1960).

[2] H. von Helmholtz, *On the Sensations of Tone* (Dover Publications, Inc., New York, 1954), pp. 80–88 and 384–387.

[3] C. V. Raman, Indian Assoc. Cultivation of Science, Bull. No. 16, 11 (1918).

reactance, and Sec. BC, except as modified by bridge impedance, is negative; together they form a series resonant circuit. For our purposes, the string may be treated as a lossless transmission line.

Bridge C is the transducer that accepts power from the string and transfers it to the body, which in turn excites the air within and surrounding it. Since the motion of the bridge in its own plane may be regarded essentially as that of a rigid body, it acts in the lower octaves primarily as a transformer.[4] Presumably its compliance is important to normal transmission at higher frequencies, though Minnaert and Vlam consider that its main function is to permit yielding to extraneous (e.g., torsional) motions.

For present purposes, it is simplest to regard the sound post as an important part of the body. It shares with the ribs the function of connecting the back to the source of vibration, and is necessary for strength; it is extremely important as a means for providing the dissymmetry needed for effective radiation, and plays a crucial role in determining the frequency and geometric form of the natural modes of the box.

The use of an enclosure—a box with vibrating walls—is an ancient device in instrument making. Even though the *f* holes of a violin were narrowed to the point of eliminating them as emitters of sound, the enclosure would still be essential since it is the variations of its volume which give the character of a simple source, these changes in volume being a difference effect between oppositely phased parts of its surface.

The bridge stands with one foot near the soundpost and the other over the bassbar. With no losses and no radiation, the bridge would see the top plate with its many vibrational modes as a complicated reactive circuit.[5] In terms of Foster's reactance theorem, there would be a series of frequencies with zero reactance, each separated from its neighbor by a frequency of infinite reactance. Losses in material and by radiation modify the reactance curve and add a curve of finite resistance. The curve, however, remains a very bumpy one. For each frequency the motional response of the body to the force exerted by the bridge is the summation of responses of the various modes. The low resistance of a resonant mode tends to "short-circuit" the others. The body thus acts like a number of series resonant circuits in parallel, as shown in Fig. 1(b), but even at resonance resistance must be several times the characteristic impedance of the string.

It does not follow that acoustical peaks must be associated with points of lowest impedance. There is at least one important exception—the air mode that interposes an impedance maximum tending to restrict bridge motion. This part of the circuit [HG in Fig. 1(b)], being described in terms of volume velocity, is shown connected with the mechanical circuits by a mechanical-to-acoustical transformer at T, with transformer ratio A to 1. Since A is different for each mode, separate transformers are shown; all these feed into the same circuit MHG.[6]

III. THE MAIN BODY RESONANCE

Meinel[7] has traced the nodal lines appearing on top and back of one good violin for the seven lowest modes of vibration. (See also reference 8.) The surface is divided with more or less clearness into many small areas at the higher frequencies, but a certain simplicity marks the lower three octaves. It is often necessary in tracing a nodal line to follow it along a plate to the edge, to cross the ribs directly or peripherally, thence along the other plate, and so forth. For each of the following resonances, 366 cps (the lowest), 690, 977, and 1380, but not at its "main" resonance at 488, the entire body was divided into only two areas separated by one endless nodal line. At the "main" resonance there were three areas. The number of areas equals the number of lines plus one. He and others have found a definite tendency for large areas to be oriented lengthwise so as to include the bass bar.

In the low octaves the restraining effect of the sound post near the right foot of the bridge leads to a greater motion at the left than at the right foot. Bridge bassbar and top plate thus rock about a nodal point near the post.

Instrument makers have commonly located the main resonance about 15 semitones above the lowest tone in the violin, 17 in the viola and cello.

The following method has been used to determine equivalent series stiffness and mass, S and M. The frequency of resonance was measured with different small loads m clamped to the bridge with mass centered at the string notch. The wolftone, if there is one, can be used as relative indicator of resonance. Since $\omega=[S/(M+m)]^{\frac{1}{2}}$, it follows by differentiation that at $m=0$:

$$M=-\tfrac{1}{2}f/(df/dm) \quad \text{and} \quad S=-2\pi^2[f^3/(df/dm)]. \qquad (1)$$

In this manner, data from Raman[9] and Saunders[10] have been used to calculate M and S for one cello and one violin:

	f(cps)	S(dyn/cm)	M(g)	$(SM)^{\frac{1}{2}}$, cgs ohms
Cello	176	1.13×10^8	92.	1.02×10^5
Violin	500	1.76×10^8	17.8	0.56×10^5.

It is interesting that the stiffnesses for these two instruments of widely different size are not very different. Such relations will be examined in Sec. VII on scaling.

This resonant circuit has been characterized by S and M, but any two of the four quantities above can be used. Violin makers, in particular those who use electronic techniques,[1] explicitly consider one of these the

[4] M. Minnaert and C. C. Vlam, Physica 4, 361–372 (1937).

[5] The mass of the bridge added to the body leads to natural modes somewhat different from those of the body alone.

[6] Strictly speaking, the radiations from plates and *f* holes should be shown as from a single simple source.

[7] H. Meinel, Elekt. Nach. Tech. 4, 119–134 (1937).

[8] F. Eggers, Acustica 9, 453–465 (1959).

[9] C. V. Raman, Phil. Mag. 32, 391–395 (1916).

[10] F. A. Saunders, J. Acoust. Soc. Am. 25, 491–498 (1953).

location of the resonance in the range of the instrument. A second is not used except as it is implicit in rules for dimensioning and in particular in the selection of wood.

The foregoing estimation of M and S is useful in dealing with a narrow band about resonance, as later in connection with the wolfnote. The stiffness measured is predominately that of the body, strings being accountable for less than 10%. The air circuit (see following section) also contributes significantly to both mass and stiffness.

The third circuit constant is resistance. It can be obtained by dividing $(SM)^{\frac{1}{2}}$ by Q, the quality factor of the resonance. Q may be found from measurements of logarithmic decrement δ, such as those by Saunders[11] ($\delta_{10}=\pi/2.30\ Q$). His values of δ_{10} range between 0.062 and 0.14. Clearly the measurement should be made for the particular instrument studied.

For the principal mode, Meinel maps the amplitude of motion over the two plates of a violin.[7] This provides data for estimating the equivalent simple source and resulting radiation. Net change in volume is equated to volume displaced by piston area A conceived as moving with the bridge-string contact; in this way one can estimate series radiation resistance referred to the same point for which equivalent mass and stiffness have been measured. The motion at the string is greater than that at the left foot of the bridge by a factor of about 1.5. From Meinel's data:

Amplitude left foot	28 μ
Amplitude bridge slot (lever ratio 1.5)	42 μ
Average amplitude over plates	7 μ
Area, two plates (ribs neglected)	1000 sq cm
Since $42A=7\times1000$, $A=$	170 sq cm.

Considering the body as a small source, radiation resistance $\pi\rho_0 f^2A^2/c$ turns out at 470 cps in this example to be 700 cgs mechanical ohms. This is a component of the total resistance to bridge motion, viz., $(SM)^{\frac{1}{2}}/Q$. With the value previously mentioned for a particular violin as the numerator, and as denominator Saunders' smallest value of Q, the total resistance is 5900 cgs ohms. Radiation resistance is therefore 12% of the total. Lacking measurements on the same instrument, we have chosen data so as not to overestimate radiation. Usually it will be considerably higher than 12%. Radiation efficiency, however, is not to be confused with over-all efficiency, which is much lower owing to inefficiency of conversion at the bow.

Similar study at other body resonances might prove interesting.

IV. AIR RESONANCE

The plate motions that produce the simple source of the previous section cause changes in the opposite sense in the air within the body. The equivalent piston area is the same. If frequency is very low, the air passes through the *f* holes without change of pressure; if high, the air is trapped and compression occurs; the cavity with its ports in this oversimplified concept is thus a parallel-tuned circuit [HK in Fig. (1b)],[8] and has an obvious similarity to reflex-bass enclosures in loudspeakers. (To avoid anachronism the comparison should be reversed.) Of the many modes possible, only the lowest in frequency is important. Similarity to a Helmholtz resonator is obvious.

However, with as peculiar a shape as that of the *f* hole and with nonrigid walls, one hardly expects to use Rayleigh's $\omega=c(d/V)^{\frac{1}{2}}$, even though he showed[12a] that with elliptical ports of small eccentricity the same frequency occurs as with a circular one of equal area. As a matter of fact, the expression yields rough estimates with *f* holes, the area of one being considered with half the volume.[13] The frequency thus calculated will be a semitone or so too low.

Along with the measurements of logarithmic decrements of body resonance, Saunders[11] measured decrements associated with the decay of transient oscillations at air resonance for many violins, old and new. The total decrement comprises components from several causes: useful radiation, surface absorption, viscosity in the air, and wall motion (loss in the wood). It would be useful to know their relative importance.

Radiation decrement. It can readily be shown that the logarithmic decrement to base 10 caused by radiation is

$$\delta_r=(2\pi^3V/\lambda^3)/2.30=27V/\lambda^3, \qquad (2a)$$

which applies only to the lowest mode, for which volume is very small compared with a cubic wavelength. Wall motion is assumed blocked.

Wall-surface loss. This is the component ascribed to the surface regarded as stationary. Absorption coefficients derived from architectural acoustics may be used. By contrast, the violin is much smaller than a wavelength at resonance, and pressure and phase are substantially the same throughout the volume. For small absorption coefficients α_w, the logarithmic decrement to base 10 of a large enclosure (all boundaries of same material) is[14a] $\delta_w=0.543c\bar{S}\alpha_w/Vf$. It can be shown that equal energy densities produce mean squares of sound pressure on the walls of violin and room which are in the ratio 1/2 to 1; decrements are in the same ratio.[15]

[11] F. A. Saunders, J. Acoust. Soc. Am. 17, 169–186 (1945).

[12] Lord Rayleigh, *Theory of Sound* (Dover Publications, Inc., New York, 1945): (a) Article 306, p. 179; (b) Article 225; (c) Article 214, Eq. (2).

[13] If the cavity were bisected by a thin longitudinal wall, there would be no effect on frequency of air resonance. Calculation indicates that frequency is lowered about one-fifth tone by the air mass added by plate thickness.

[14] L. L. Beranek, *Acoustics* (McGraw-Hill Book Company, Inc., 1954): (a) p. 305; (b) p. 300.

[15] The reason becomes apparent on consideration of any oblique mode in a rectangular space in comparison with the (0,0,0) mode, an approximation to which can be realized by adding a port, as in the Helmholtz resonator. In the zero mode, p^2 averaged along an edge exceeds that for an oblique mode by one factor of 2, over the walls by two factors of 2, and over the volume by three such factors, since the average value of cosine squared is $\frac{1}{2}$. Decrement is proportional to p^2 averaged over the walls (rate of dissipation), divided by p^2 averaged over the volume (energy storage). This adds a factor of 4/8, or $\frac{1}{2}$, to the ratio of the decrement for the zero order divided by that for the oblique mode.

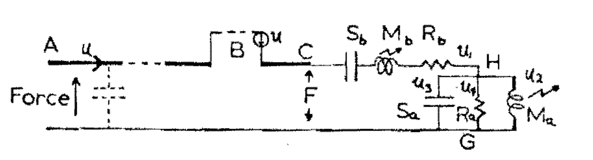


FIG. 2. Simplification of Fig. 1(b), showing air resonance and one body resonance.

Hence wall-loss logarithmic decrement in the violin is

$$\delta_w = 0.027c\bar{S}\alpha_w/Vf = 0.027\bar{S}\alpha_w\lambda/V. \tag{2b}$$

Viscosity in the air. It is assumed that with ports as wide as the *f* holes in the standard violin, this is a negligible effect.

Loss within the wood. In the simplified circuit of Fig. 2, a transient in HG would suffer loss from resistance R_b unless wall motion were blocked during measurement. Calculation suggests the effect to be appreciable but possibly not large. However, it seems desirable to exclude from the definition of this decrement effects of important circuit elements such as R_b, which are shown explicitly. One would include effects of modes, if any, that are not directly excited by the bridge.

Discussion. The losses contributing most to the decrement of the air circuit as thus defined seem, therefore, to be wall-surface loss and radiation. For a given frequency a change in volume affects these components oppositely, one varying inversely and the other directly. Account also must be taken of the fact that $\bar{S}$ may be a function of volume. A question often raised concerns the best height of the ribs, dimensions of plates being determined by other considerations. In Fig. 3 are plotted computed values of these components and their sum. Account was taken of the fact that $\bar{S}$ includes the area of the ribs. Resonance frequency (wavelength) is held constant by changing width of *f* holes or by use of additional ports. The α_w used is 0.04, the value for wood floors on solid foundation,[14b] architecturally a small value.

For assumed values of parameters, the two components are equal for a rib height of 5 cm. Their sum has a minimum between 3 and 4 cm. This may be compared with the rib height used by Stradivarius, 3.0 cm. It is true that the minimum is not a sharp one, and that its calculated value will shift somewhat with better data. Nevertheless, the agreement seems significant, as does the consistency with Saunders' data. His median value of 0.115 contains a significant amount resulting from wall motion. The value shown by Fig. 3 is 0.105. It would, of course, be a mistake to lose sight of the considerable spread from instrument to instrument—from 0.09 to 0.14—and the fact that the computation depends on choice of particular values of α_w and f.

Other air modes. Because air resonance is important in the lower register, it is sometimes supposed that the many natural modes of the cavity must play an important part in the upper ranges. It is argued that the volume is large enough to support dozens of modes within the range, and that these must be helpful. The experimental evidence, however, is that they are of little or no value. Saunders[10] reported that "there appeared to be upper resonances near 1300, 2600, and 3660" in a certain violin, but his general conclusion was that the output from the *f* holes is unimportant except near the lowest resonance.

To be effective a natural mode has to satisfy two requirements: it must be energetically excited by the walls, and it must be "impedance-matched" to the *f* holes. The lowest mode satisfies both by design. It is excited because the wall motion provides the net changes in volume[7,8,16] required to induce "zero order" pressure changes. Secondly, its very existence depends upon there being an air flow through the *f* holes, that is, *upon radiation.* By contrast consider the other modes: their geometric structures, except by accident, are unrelated to those of the wood, and there is no obvious basis for expecting power transfer to the air within. Moreover, such excitation as may fortuitously occur will not necessarily cause the breathing through the *f* holes needed for radiation; the holes are apt to be too small or too large or in the wrong place.

V. COMBINATION OF AIR AND BODY MODES

Air and main body resonances are not isolated means, but in good instruments are matched for best total effect to insure a strong lower register. In studying circuit behavior, they need to be considered together. The problem, which is complicated, will here be limited to simple conditions.

To this end, consider the circuit of Fig. 2 in which one resonant body mode is assumed to predominate, its series circuit being in series with the shunt resonant circuit representing the air circuit, and its impedance being sufficiently great to control wall velocity.[17] Here the acoustical circuit HG has been transformed into its mechanical equivalent in terms of linear velocity at C, the top of the bridge. Assuming the configuration of the body mode to be independent of frequency, radiation resistance of the plates (without contribution from the *f* holes) is proportional to frequency squared.

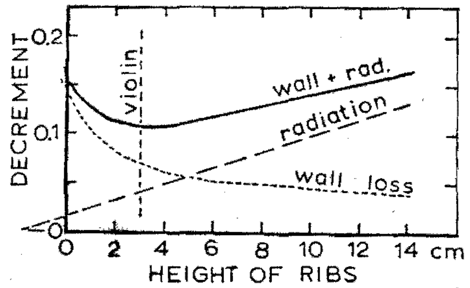


FIG. 3. Two main components of air decrement.

[16] H. Backhaus, Z. Physik 62, 143 (1918); 72, 218 (1931).
[17] A more refined calculation would forego the last assumption. The impedance measurements that Eggers[8] made on a cello are of interest in this connection.

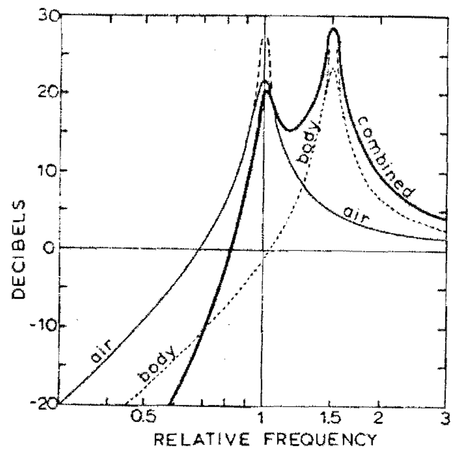


FIG. 4. Relative sound pressure, air resonance, and one body resonance. In the violin other peaks occur at the right.

Following the method of Appendix I, relative performance is calculated for the following typical condition:

Air resonance at relative frequency	1.0
Q_{air} (mean of Saunders' values)	=12.
Body resonance at relative frequency	1.5
Q_{body} (Saunders' mean)	=14.5.

The dotted curve of Fig. 4 depicts radiated response without air resonance, a condition that might be realized by using very large volume or by closing the *f* holes; the violin now has the advantage of enclosure, but not of air resonance. If resonance is now permitted and adjusted to an interval of a fifth (frequency ratio of 3/2) below body resonance, the light solid line gives the improvement in decibels. Finally, the combination is shown by adding the two, giving the heavy solid line.

The "air" curve has several features to notice. Most prominent, of course, is the resonance peak. At lower frequencies the air advantage falls off until, at relative frequency 0.7, half an octave, it becomes zero. Within practical bounds, a difference of Q (or δ) would not affect this conclusion, though it could change response radically within 10% of resonance. In violas air resonance is not infrequently placed eleven semitones (frequency ratio 15/8) above the lowest fundamental, which consequently suffers an "air-resonance disadvantage" of 8 dB, though the second and third harmonics can be in a very strong position. The curve is somewhat too optomistic at its high-frequency end because of oversimplification in circuit representation. It seems safe to say, however, that there is some gain over an entire octave and 9 dB or more over half an octave.

The curve for body resonance alone, instead of falling to very low values at high frequencies, approaches a horizontal asymptote, a point of considerable significance. This behavior, which holds for all modes above resonance and which has its counterpart in direct-radiation loudspeakers, is the result of two opposing tendencies. The conformation of a mode and, therefore, its equivalent mass are taken to be independent of frequency; hence, the velocities set up by a given applied force vary *inversely* with frequency. But the sound pressure radiated by a given velocity is *directly* proportional to frequency and thus annuls the effect of the decline of velocity. The way in which the various modes will combine depends upon the phase relations of their simple sources; it is plausible to suppose, however, that the lower modes thus provide a more or less level table land on which the higher ones erect their peaks, and that this is an important contributor to violin tone.

Another result of Appendix I is an expression for the ratio of sound pressures produced at body and air resonances:

$$p_{body}/p_{air} = (f_b/f_a)^2(Q_b/Q_a). \qquad (3)$$

This agrees reasonably well with ratios found from single-frequency curves by Saunders (see reference 11, p. 173).

How important are the Q's? In Fig. 4 the effect of an increase in either Q beyond usual values produces an elevation of level within a very narrow band at resonance, an effect as apt to be harmful tonally as helpful. There seems little to seek in a Q higher than that which gives a 3-dB bandwidth of one tone—a Q of 8 or 9—and there may be something to avoid in bandwidths that can be straddled by adjacent semitones. The importance of resonances, in other words, is to provide broad foothills rather than sharp peaks.

What is the best volume to be used? It is significant that in approximate Eqs. (1), (2), and (3) of Appendix I, volume does not appear as an explicit term affecting radiation. It is true that resonant impedance of the shunt circuit increases as volume decreases, suggesting an advantage because more power can be abstracted from the constant velocity source. But this is opposed by reduction of radiation decrement and, therefore, of radiation efficiency, leaving a change of only second-order importance.

There is another consideration, however. In good instruments the "piston area" A (see Sec. III) is made as large as possible, so large in fact that the impedance offered to the motion of the top plate by air resonance is by no means negligible. The sign of its reactance component at a frequency somewhat below resonance is positive, so as to tend to cancel the negative reactance of the body. If the volume is now made too small, this cancellation can be complete, so that the impedance into which the string works is a relatively low resistance capable of interfering with the normal operation of the bow. Something of this sort seems to be the reason for the airtone "wolfnotes."[18]

[18] The behavior and elimination of airtone wolfnotes have been studied by F. A. Saunders (private communication). Impedance measurements by Eggers,[8] Fig. 18, are of interest in this connection.

(45)

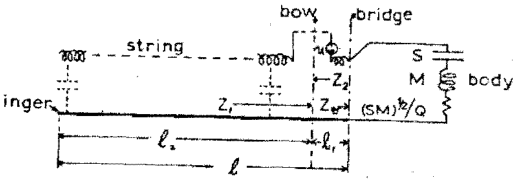


FIG. 5. Elements of wolfnote circuit.

VI. THE WOLFNOTE

The most troublesome wolfnote, however—a cyclic stuttering response to the bow on the heavier strings, particularly in cellos—can occur when the fundamental is within a half-tone or less of the main resonance; it may occur in otherwise fine instruments. Its behavior immediately suggests beating and coupled circuits. The best explanation has been one published by C. V. Raman 45 years ago.[3,9] Having much in common with his theory, the present one, which is stated in the language of circuits, differs in one important respect.

Through most of the frequency range the impedance presented by the bridge is high compared with the characteristic impedance of the strings, perhaps ten times or much more. Trouble may ensue at resonance when this ratio is well below ten, and the Q is in the range found by Saunders.

In this study it is the impedance presented to the bow that is the most informative. Calculations must take account of the distributed nature of the mass and compliance of the string, hence requiring standard methods of computation for transmission lines, as indicated in Fig. 5 and Appendix II. Resistance in the string itself is neglected.[19] For generality, equations are written in terms of dimensionless ratios: impedance relative to K, the Q of the bridge circuit, Ω and Ω_s, and the fractional part of the string length between bow and bridge. Since wolfnote is not sensitive to the latter, two parameters Q and $K/(S_bM_b)^{\frac{1}{2}}$ remain as the fundamental data prescribing circuit behavior when impedance seen by the bow is considered as a function of frequency.

In the normal situation, the string presents to the bow an unambiguous impedance; it is that of a simple series tuned circuit having positive resistance low enough to be matched by the negative of the bow-string contact, and reactance that passes through zero at the frequency of operation as shown for the fundamental by the broken line in Fig. 6(b). Though still not a simple problem, in view of the complicated impedance pattern in which the various components must seek a compromise frequency, the situation contains no obviously tempting invitation to misbehavior. By contrast, consider the impedance pattern at the main resonance as shown in Fig. 6, in which for a given string length (e.g., $\Omega_s = 1.028$) there is not one but three frequencies at which reactance is zero. Steady oscillations can conceivably occur at any of these frequencies.[20] If the negative resistance is insufficient to cope with the high resistance at the point of tuning, it may, nevertheless, be adequate at the two outside points. That is to say, if bow pressure is insufficient for the normal vibration, it still may be enough for the outside pair because of their lower resistance. Let these frequencies be $(\Omega'+\Delta/2)$ and $(\Omega'-\Delta/2)$. If amplitudes are equal, total string velocity will be proportional to

$$\cos 2\pi(\Omega'+\Delta/2)+\cos 2\pi(\Omega'-\Delta/2) = 2\cos 2\pi\Omega' \cos\pi\Delta t, \quad (4)$$

the familiar expression for a beat. Speaking in terms of "instantaneous" frequency instead of Fourier components, this is a wave of average frequency Ω' pulsating at frequency Δ. Frequency is always Ω', but there is a phase reversal in passing through zero amplitude.

Raman[9] criticized G. W. White's suggestion[21] that it is a beating process on the ground that one of the frequencies must be that of a free oscillation that will soon decay. I believe the conclusion incorrect that there is no beating. The two oscillations suggested here are equally forced.

These two oscillations (referred to as the "fundamen-

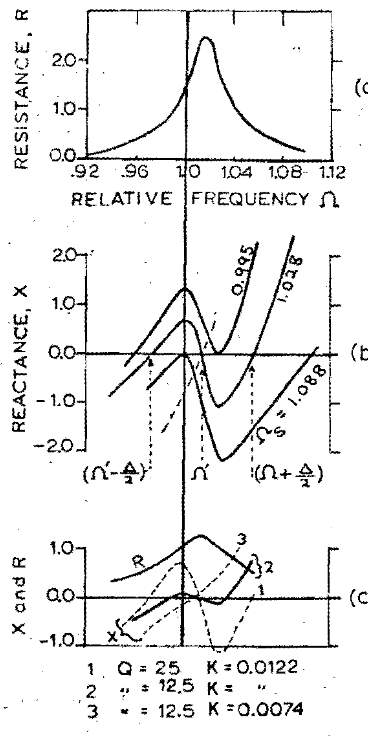


FIG. 6. Impedance seen by bow (Z_1+Z_2 of Fig. 5); (a) and (b) for a bad wolfnote; in (c), 1, 2, and 3 show consecutively improved conditions.

[19] H. Backhaus, Z. Physik 18, 98 (1937).

[20] Even though the negative sign of reactance slope raises a question of stability, with the highly nonlinear behavior of the bow-string contact, the stabilizing effect of harmonics on a string free of phase distortion will in borderline cases probably permit oscillations in spite of the slope.

[21] G. W. White, Proc. Cambridge Phil. Soc. 18, 85–88 (1915).

tal pair") are not the only sinusoidal components that move through the whole beat cycle substantially unchanged. The same is true of the even harmonics of Ω'. Raman showed that the octave becomes prominent at the beat minimum. Curiously enough, this prominence is not because the even terms have grown, but because the odd terms have subsided; the sawtooth at its maximum amplitude contains even terms of about the same amplitude and phase as at the minimum. This is brought out clearly and simply by a graphical separation of odd and even terms in the wave shown in Fig. 7(a).[22] The maximum displacement of the even components shown in Fig. 7(b) is about half that in Fig. 7(a), and the slopes of the long sections are the same, matching the same bow velocity. Addition of Figs. 7(b) and 7(c) shows how the intermediate discontinuities have been cancelled by the odd terms of Fig. 7(c). Removal of the odds will, therefore, bring the "octave" back.

An important difference between a linear and a nonlinear generator needs mention. In the former it is possible for oscillation to occur at one of the fundamental pair alone. The principle of superposition gives them complete independence. But with bowing, the recurrence rate depends by virtue of the necessary harmonics on the string length and the string's simple phase characteristic. Neither of the pair can exist without the other because its frequency is so different from a recurrence rate possible on the string. On the other hand, by cooperation they can produce an instantaneous frequency acceptable to the string, equal to half the trigger rate of Fig. 7(b). Like Siamese twins, they can exist as a pair, but not otherwise.

If this theory is correct, it should be possible from experimental evidence to show that an epoch of maxima has undergone a phase shift of π relative to the preceding one. This can, in fact, be seen in Raman's oscillograms (Plate I of reference 3), which show simultaneous motion of bridge and string, that of the string being suggested in Fig. 8. At the epoch where the amplitude of the bridge motion is growing most rapidly, the string, as Raman indicates, has a clean sawtooth displacement. At this moment bridge amplitude is matched to bow velocity and pressure. The amplitude of bridge motion continues its growth for a time, but the sawtooth shows signs of deterioration in the form of new discontinuities midway between those of the series just considered.

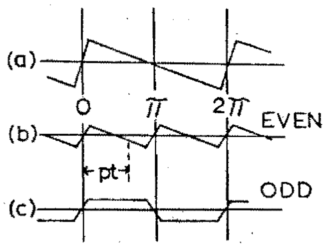


FIG. 7. Separation of even and odd terms of idealized string displacement.

[22] Odd components of $f(pt)=[f(pt)-f(pt+\pi)]/2$; even components$=[f(pt)+f(pt+\pi)]/2$.

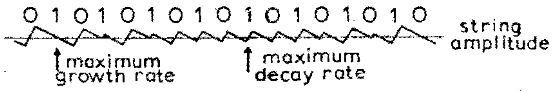


FIG. 8. String displacement through one wolfnote beat.

This new series is destined to be the sole series of discontinuities during the next period of bridge-motion growth. To see this in the original oscillograms requires close examination, preferably with a magnifying glass,[23] but it is readily followed in Fig. 8. If we place a zero adjacent to each clearcut discontinuity at the left and a 1 midway to the next later discontinuity, and continue this alternate naming through the octave period to the next clearcut stage, we find that the discontinuities of the latter are named 1, not 0. All three transitions shown in the original give the same result. This indicates a phase shift of 180° in instantaneous frequency in passing through the minimum, and this, of course, is what a beat requires.

The bad situation shown in Figs. 6(a) and (b) is greatly improved in Fig. 6(c) by reducing Q from 25 to 12.5. Though the S shape remains, the reactance curve is nearly flat in the region of interest, the fundamental pair closer together, and their resistances just slightly less than at midband. It is doubtful that a wolf could occur. Curve (3) shows the further advantage of reducing characteristic impedance of the string through reduction of weight. The side frequencies have now disappeared. That light strings' help is, of course, well known.

The rate at which beating occurs is consistent with the difference in frequency within the fundamental pair. Applying Fig. 6(c) to the cello, the indicated rates for conditions (1) and (2) are, respectively, 16 and 8 per sec. The delays of bridge maximum with respect to string maximum are calculated as 0.36 and 0.17 of a beat cycle for the same respective conditions, agreeing with Raman's[9] 0.25, which would have been found here had Q been taken at the more typical value of 17.5.

Finally it should be re-emphasized that conditions at harmonic frequencies may have some connection with wolfnote, and that (1) the distance of the strings from the nodal line about which the bridge swings and (2) the angle of bow motion certainly do have an effect. One experimental condition was to bow the cello C string (the lowest) *underneath* rather than over the strings, with hand held as high as possible without the bow touching the wood. On this most "wolfish" of strings, the wolftone, as expected, disappeared.

Wolftone criterion. It is desirable to show graphically the relations prevailing under different conditions of susceptibility to wolftone. To this end Fig. 9 provides dimensionless coordinates on which can be exhibited essential parameters applying to all instruments of the violin family, viz., $K/(S_bM_b)^{\frac{1}{2}}$ and logarithmic decre-

[23] The reproductions in a communication to *Nature* are not clear enough.

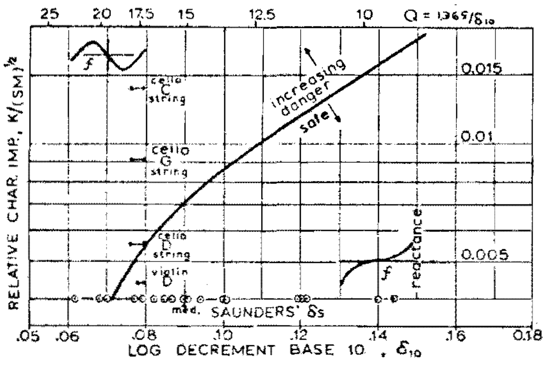


FIG. 9. Wolfnote criterion.

ment. The heavy line shows the locus of conditions for which the midpoint of the *S* curve in Fig. 6 is horizontal, i.e., on the verge of having three intersections; it is the boundary between safe and questionable conditions.

On the basis of data previously cited for one cello and Saunders' median δ, points are plotted for the three strings that are used to produce the frequency of main resonance. For the cello D string, the point falls close to the safe area as one expects from experience. For the heavier G string, the point is definitely away from the "safe" region, also in agreement with experience; being much used in this position, this string causes the cellist the most annoyance. In going to the C string, one would expect and one finds that it may be very difficult indeed. Fortunately this note is played on this string relatively infrequently. The point representing a violin indicates relative freedom from wolftone.

Eliminating the wolftone. The instrumentalist is rarely interested in an explanation. What he wants is to have the curse removed. The idea occurred to the author and to others[8] to insert a narrow-band suppressing circuit in series with the bridge motion. This is most conveniently done by attaching to one of the strings between bridge and tailpiece a mass of a few grams chosen to tune the string end to the wolfnote. On the cello tuning can be done by ear by tapping the load with the eraser of a pencil used endwise. Without tuning there is no guide to adjustment. A second precaution, not always necessary but to be recommended, is use of a suitably lossy or nonringing substance such as the rubber of a large pencil eraser. Calculation indicates a desirable Q to be 10 or less, so as to give a wider band than the main resonance. The less obstinate the wolf, the farther the load may be placed from the bridge. Hutchins finds molding clay very convenient, since it has desirable loss and its mass is easy to adjust.

Avoidance of wolftone by means external to the body may give more freedom to the maker in other respects, such as ability to use wood of high c/ρ (see Sec. VIII).

VII. DIMENSIONAL SCALING

The process of tuning a violin includes more than adjusting string tension; the luthier must first "tune" its pattern of wood resonances with respect to those of string and air. If he is designing an instrument to occupy a new frequency range, all of these resonances are changed with respect to those of instruments that have become conventional to him. If he wishes to change size without changing frequency range, either to accommodate smaller hands or to allow larger ones to work to better advantage, an understanding of the principles governing modified dimensional scaling will be helpful.

Complete scaling is theoretically no problem; all that is needed is thoroughgoing change of all dimensions in proportion to change in air wavelength, and use of identical materials. (Actually the difficulties of even one-to-one scaling—that is, copying—are considerable, partly because of unavailability of identical materials; this ever-present problem is not considered here.) In general, complete scaling is not practicable because of the impossible demands that it places on strength or size of the player, or for other reasons. Failure to understand scaling leads to errors in construction; for example, in making a viola[24] luthiers have sometimes chosen a plate thickness in proportion to that of a violin; however, this is too much.

Before undertaking to scale, it is necessary to decide on basic aims and the compromises one is willing to accept. Discussion here will be limited to conventional instruments; for concreteness we shall rather arbitrarily regard the violin as the most suitable starting point, and ask what this leads one to expect about the viola and cello and their strings. There are three basic ratios to consider:

(1) The lowest frequency of the "new" instrument is to equal that of the violin divided by ϵ. That is, air wavelength is multiplied by that factor. For the viola $\epsilon = 1.5$, for cello 3.0.

(2) The instrument being thought of as lying on its back, its shape as seen from above, is similar to that of the violin, and all horizontal dimensions are multiplied by σ. For a $16\frac{1}{2}$-in. viola $\sigma = 1.17$, for a typical cello 2.1.

(3) The pattern of body resonances is to remain the same logarithmically, and wavelength of the main resonance is to be multiplied by η. Usual practice places η at 1.33 for the viola and 2.66 for the cello.

These factors are defined so as to be greater than unity in going to viola or cello. Plate shapes not being strictly the same, σ involves a small compromise.

In a completely scaled instrument, the air resonance takes care of itself. Incompleteness will be evident in width and thickness of ribs and width of *f* holes. It seems desirable to scale length of *f* holes. Ribs can be scaled according to principles of air resonance (Sec. IV).

A reservation with respect to the arch needs mention. The rigidity of a plate depends on (1) its stiffness in flexure and (2) its two-dimensional curvature. With a flat plate only the first enters. (The effect on resonances

[24] I am indebted to C. M. Hutchins for information on this and other matters.

is somewhat analogous to that in a piano string where two forms of stiffness, tension, and rigidity occur.) While our analysis in strictness, therefore, applies only to a flat violin (thickness graded, however), it is believed to give numbers of significance even considering the arch.[25]

For a flat plate of arbitrary shape and graded thickness, the basic relation is that the frequency of a flexural mode is directly proportional to the scale of thickness and inversely to the square of a horizontal dimension. We assume that the main effect of the ribs, aside from providing enclosure and coupling between plates, is that of mass loading at the edges, which are by no means immobile.[7,8] The idea that, in scaling, total mass of ribs should remain proportional to mass of plates is suggested by the fact that in the cello they are proportionately thinner than in the violin, their height being proportionately greater.[24]

Thickness. Subscripts 1 and 2 refer respectively to the model and the "new" instrument. H is thickness at some reference point. We now have

$$f_1=kH_1/l^2 \quad \text{and} \quad f_2=kH_2/l^2\sigma^2, \tag{5}$$

l being length, and since $f_1/f_2=\eta=H_1\sigma^2/H_2$,

$$H_2/H_1=\sigma^2/\eta. \tag{6}$$

Stiffness. Consider design in two steps—first to an intermediate instrument completely scaled by factor σ and indicated by primed symbols. A general relation for stiffness S of dimensionally similar shapes is

$$S'/S_1=\sigma. \tag{7}$$

Altering intermediate to new, change of thickness affects stiffness in proportion to the cube, or

$$S_2/S'=(H_2/H')^3=(H_2/\sigma H_1)^3; \tag{8}$$

and from (6) and (7),

$$S_2/S_1=\sigma^4/\eta^3, \tag{9}$$

the ratio of stiffness entering into corresponding resonances.

Mass. Corresponding masses are proportional to total mass of the plate:

$$M_2/M_1=\sigma^2.\sigma^2/\eta=\sigma^4/\eta. \tag{10}$$

[25] The tendency for deformations to be predominately inextensional (potential energy purely flexural) is discussed in Rayleigh's *Theory of Sound*, Article 235 b. The fact that the arch does have an effect in acoustical behavior has recently suggested that similarity of behavior should result if in scaling we maintain the same ratio between flexural and extensional potential energies. An elementary analysis indicates that this is obtained simply by scaling altitude of arch according to σ^2/η, the same factor used in Eq. (6) in scaling thickness; that is, by holding constant the ratio, arch/thickness, rather than the more naturally used ratio, arch/horizontal dimension. Since arch is a means of adjusting prominence of high frequencies with respect to lows, and since low-voiced instruments do not necessarily require the same balance as high-voiced, exact scaling of the arch may not be the best.

Body impedance.

$$(S_2M_2)^{\frac{1}{2}}/(S_1M_1)^{\frac{1}{2}}=\sigma^4/\eta^2. \tag{11}$$

Ultimate strength. Ratio of ultimate strength of forms completely scaled is

$$Y'/Y_1=\sigma^2, \tag{12}$$

and if as in the flexural strength of isotropic materials,

$$Y_2/Y'=(H_2/H')^2=\sigma^2/\eta^2, \tag{13}$$

$$Y_2/Y_1=\sigma^4/\eta^2. \tag{14}$$

This is the same as for body impedance, Eq. (11).

Strings. General opinion, borne out by the need of old instruments for stronger bass bars, is that string tension T with a factor of safety is set with relation to ultimate strength of the body.[24] Therefore, from Eq. (14)

$$T_2/T_1=\sigma^4/\eta^2. \tag{15}$$

It follows from Mersenne's law for strings that

$$\mu_2/\mu_1=\sigma^2\epsilon^2/\eta^2. \tag{16}$$

From (16) and (15) ratio of characteristic impedance is

$$K_2/K_1=\sigma^3\epsilon/\eta^2. \tag{17}$$

"Wolf ratio." Finally taking ratio of Eqs. (17) and (11), we have

$$[K_2/(S_2M_2)^{\frac{1}{2}}]/[K_1/(S_1M_1)^{\frac{1}{2}}]=\epsilon/\sigma, \tag{18}$$

expressing the relative impedance positions in Fig. 8.

In order to check these relations experimentally, instruments compared must embody the uniformities of construction assumed. In individual instruments this is difficult assurance to gain. However, certain general relations may be tested. Thus, Eq. (6) indicates why in violas and cellos thickness is less than for complete scaling since $\eta>\sigma$. Again according to Eq. (15), one expects cello-string tension to be 2.7 times as great as in the violin ($\sigma=2.1$, $\eta=2.7$); for two sets of strings compared, the ratio was 2.4.

Equation (18) is particularly interesting since it explains why wolftone trouble increases in going from violin to viola to cello; the reason is simply that the sizes of the instruments have not been increased in proportional to the air wavelength, and maximum safe tension has been insisted upon in the strings. The chronic susceptibility of the cello to this trouble is the price paid for the convenience of a small instrument, small compared with one completely scaled.

Since *with scaling* A is proportional to σ^2, and f at body resonance to $1/\eta$, radiation resistance that depends on A^2f^2 is proportional to σ^4/η^2. This is identical with the expression that Eq. (11) gives for body impedance. It follows that a body-resonance radiation decrement, which is the ratio of these quantities, is invariant. If it is true that body losses depend primarily on the wood, we may conclude that total decrement is also invariant in the scaling process, since Rohloff finds the decrement of wood to be independent of frequency.[26] Within these limits, this justifies the name "wolf ratio" used with Eq. (18). In absence of scaling, we require measurement of both decrement and $K/(SM)^{\frac{1}{2}}$ for the application of Fig. 9 to individual instruments.

VIII. ON THE REQUIREMENTS OF WOOD

Violin makers have always attached great importance to the selection of wood, not only as to species but also the characteristics of the particular piece to be used. Acousticians have measured elastic properties, density, and damping coefficients, and the more scientifically minded makers are trying to take advantage of such procedures. It is important to relate these measurements to the luthier's problem in as simple a manner as possible.

An important question is: When are two pieces of wood acoustically equivalent? Should one try to match both the elastic modulus and the density?

Along and across the grain elastic properties are different velocities of compressional waves appearing to be in the order of three or four to one. We assume a fixed ratio and consider the elastic behavior determined by a single modulus E, density, and scale of thickness. As in the previous section, flat plates of graded thickness are assumed.

Flexural similarity and c/ρ. Having given a reference plate (subscript zero), let it be required to duplicate its acoustical behavior in a plate of different material (subscript 1). Consider any point and a line through it in the plane of the plate, the line being part of the linear wavefront of a flexural wave. Such a wave is characterized by a torque per unit length lying along the front and traveling along with it. This torque is directly related to the potential energy of the medium, which is momentarily located in the stiffness at the point under consideration. The kinetic energy is similarly associated with the transverse velocity of the mass per unit area (rotational energy being negligible in the thin plates of the violin). Flexural behavior is similar in the two plates when their stiffnesses per unit length and densities per unit area are equal. Stiffness per unit length is proportional to E and to the cube of thickness (as in the analogous static problem of the beam). Hence,

$$E_0H_0^3=E_1H_1^3. \qquad (19)$$

Equal densities per unit area require that $H_0\rho_0=H_1\rho_1$, or

$$H_1/H_0=\rho_0/\rho_1. \qquad (20)$$

With $(E/\rho)^{\frac{1}{2}}=c$, it follows that

$$c_1/\rho_1=c_0/\rho_0. \qquad (21)$$

This means that *even if we are not able to duplicate c and ρ separately, reactive behavior remains the same if their ratio remains invariant.* However, ratio of thickness must change as required by Eq. (20).

[26] E. Rohloff, Ann. Physik, No. 5, 38, 177–198 (1940).

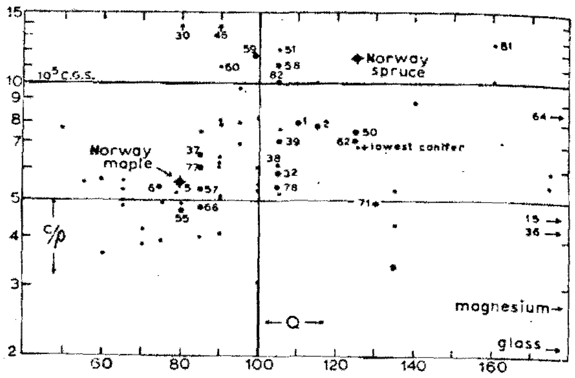


FIG. 10. c/ρ vs Q for various species of wood (based on Barducci and Pasqualini).

c/ρ and circuit parameters. The ways in which c, ρ, and c/ρ enter into circuit relations may now be indicated. Consider a violin plate of standard shape (width vs length) and assume it distorted in the pattern of a tap-tone vibration. A stiffness and a mass will be involved whose changes with certain variables are indicated by the following proportions:

Stiffness $S\propto EH^3l^{-2}$

Mass $M\propto \rho Hl^2$

Tap-tone frequency $f_b\propto (S/M)^{\frac{1}{2}}\propto cHl^{-2}$

$$\text{Thickness } H\propto (1/c)f_bl^2 \qquad (22)$$

$$\text{Impedance } (SM)^{\frac{1}{2}}\propto (E\rho)^{\frac{1}{2}}.H^2\propto (\rho/c)f_b^2l^4 \qquad (23)$$

$$\text{Mass } M\propto (\rho/c)f_bl^4 \qquad (24)$$

$$\text{Stiffness } S\propto (\rho/c)f_b^3l^4. \qquad (25)$$

Since f_b and l are determined by considerations unrelated to the wood, plate impedance mass and stiffness depend on the wood only through the parameter c/ρ, thickness on c.

Comparison of species. Barducci and Pasqualini[27] measured c and ρ for 85 species of wood. In Fig. 10 their data have been adapted to display the violin-wood parameter c/ρ as a function of its Q. Most of their species are plotted, but to avoid confusion only those represented by four or more specimens and indicated by black circles are numbered. (Numbers give the order in their table.)

A fact immediately evident is that *Picea excelsa* (the spruce which in Europe has traditionally been used in top plates) is high, whereas *Acer platanoides* (the maple used in backs) is low. The former has few neighbors to be candidates for substitution, the latter several. For reasons to be mentioned, horizontal separation of points is difficult to interpret. Comments largely drawn from Howard's *Timbers of the World* (MacMillan and

[27] I. Barducci and G. Pasqualini, Nuovo cimento 5, 416–446 (1948).

Company Ltd., London, 1951) are listed below for some neighbors of *Picea excelsa*.

#58	*Populus alba*	Close, hard, tough texture. Sounds promising.
#60	*Populus nigra*	Same characteristics as #58.
#59	*Populus canadensis*	
#51	*Pinus cembra*	Knots prevalent, otherwise promising.
#81	*Thuja plicata*	A cedar. Perhaps good, except for splits and shakes.
#82	*Tilia europaea*	A linden. Sounds good.

The craftsman may rule some of these out for nonacoustical reasons.

Relation of top and back. Measurements by Meinel (Fig. 16 of reference 7; also reference 8) show that at main resonance (and presumably generally) the back contributes materially to the equivalent simple source of the violin. Though the top is the more important as radiator, the back can by no means be neglected as a contributor; and the proper matching of one to the other seems essential to insure a strong, simple source over the frequency range. The conjecture, therefore, appears justified that the relation of impedances of the plates [Eq. (23)] needs to be maintained by keeping the values of c/ρ in the same approximate ratio of 2 to 1, which practical experience has led to, and which Fig. 11 shows for *Picea excelas* and *Acer platanoides*.

Other values of c/ρ.[23] On the assumption of some such balance between top and back, what is to be expected when, instead of using the customary spruce, the c/ρ of the top of a new violin is made lower, as it sometimes is, by using woods lower in Fig. 10? Equation (23) indicates that its body impedance will be increased. The oscillating force that a string is able to deliver to the bridge for a given amplitude of string motion is proportional to tension. Hence with unchanged strings the velocity produced in the radiating surfaces and hence the sound pressure will be correspondingly lower. The extent to which this disadvantage can be overcome depends on the willingness of the performer to use heavier strings and the ability of the structure to withstand the greater load. These considerations make it obvious why maple would be a very bad choice for the top plate. The 2-to-1 impedance increase that was mentioned in the previous paragraph would cause a 6-dB loss unless possibly cancelled by doubling string tension, and it is not obvious what material having a still lower c/ρ would be suitable for the back. The weight of the box would be almost doubled.

Oppositely, higher ratios make more power available and deserve careful trial. However, the problem is not simple. The same lowering of impedance which adds to acoustical output increases vulnerability to wolfnote. Though probably acceptable in a violin, this might be serious in a cello unless a wolf eliminator is used. Strength of wood is another consideration. When string tension is proportioned to strength of structure, it appears that a different wood parameter, $\phi/(c\rho)$, measures relatively the upper limit of sound pressure produced. Here ϕ is the strength function of the wood (e.g., bending, shear, tension across grain, etc.), in which the structure is most vulnerable as used in the violin.

Damping requirements of wood. In measurements of elastic characteristics of wood, the usual emphasis on properties along the grain has led to a preponderance of values quoted[27] between 60 and 130, whereas the Q of the principal body resonance of the assembled instrument[11] ranges from 10 to 20. It has, of course, been appreciated that cross-grain Q's are the lower by a factor near 4 and must certainly depress the resultant. The difficulties of the problem have precluded an estimate of what resultant Q one should expect from a plate.

There is one qualitative consideration worth mentioning; namely, the relative narrowness of the instrument and its vibrational patterns in comparison with length, and the resulting tendency to emphasize the effect of cross-grain constants, both as regards to potential energy of vibration and energy loss.

Consider a flat, rectangular plate of wood "supported" (hinged) along its edges and vibrating in its lowest flexural mode.[12b] We may suppose that if ratio of length a to width b equals that of length to average width of the body of the violin, the energy relations will be somewhat comparable. At the center of the plate, the principle curvatures lie parallel and perpendicular to the grain, and, if we ignore Poisson's ratio, the potential energy of deformation[12c] per unit area is $W\propto EH^3(r_a^{-2}+r_b^{-2})$ for an isotropic material, r being radius of curvature. For wood (anisotropic), with $c=(E/\rho)^{\frac{1}{2}}$ and, for a given s, $r_a\propto a^2$, and $r_b\propto b^2$,

$$W\propto s^2H^3\rho(c_a^2a^{-4}+c_b^2b^{-4}). \qquad (26)$$

Here the first term corresponds to the long dimension a, the second to the short one b. It is immediately evident that although c_b in wood is much smaller than c_a, the inverse fourth power of width can be a powerful influence in emphasizing cross-grain energy if width is, in effect, much smaller than length.

Similarly, it follows that rate of energy loss at the center is

$$dW/dt\propto s^2H^3\rho(c_a^2a^{-4}Q_a^{-1}+c_b^2b^{-4}Q_b^{-1}). \qquad (27)$$

The cross-grain term is now further emphasized relative to the other by the inverse of its smaller Q. We should, therefore, expect the Q of a violin (corrected for radiation resistance) to lie nearer to that indicated by cross-grain wood samples than to that obtained with samples cut along the grain; that is, nearer to some compromise value between 30 for the spruce plate and 20 for the maple, than to a compromise between 125 and 80.

[23] Gleb Anfiloff, Znatie-Sila (February 1961). According to a summary by G. Pasqualini, the importance of c/ρ was emphasized by Mr. Anfiloff.

The effect of the arch may be to lower the Q still further.

It may be, therefore, that simple wood loss and radiation are enough to account for the low Q's of violins. The very fact that these instruments are built-up structures may accentuate unfavorable strains not as yet sufficiently studied, perhaps such as shear along the grain. Other mechanisms for increasing losses, of course, deserve consideration, such as pre-stressing of bassbar and top-plate, as proposed by Rohloff.[26]

ACKNOWLEDGMENTS

It is a pleasure to express my indebtedness to Professor F. A. Saunders for the benefit of his long and fruitful experience in this field, and to Mrs. Carleen M. Hutchins for her insight into the problems of scientific violin making.

APPENDIX I

Refer to Fig. 2, right half, beginning at C where force F is applied by string. Subject to approximation that impedance of air circuit is negligible compared with reactance of body,

$$|u_1|^2 = F^2/\{s_b M_b[1/Q_b^2 + (\omega/\omega_b - \omega_b/\omega)]\}. \quad \text{(A1)}$$

Current-producing radiation is $(u_1 - u_2)$ (external surfaces and f holes) and equals $(u_3 + u_4)$. Its radiation resistance is that of a simple source:

$$R_r = k\omega^2, \quad \text{(A2)}$$

where $k = \rho_0 A^2/4\pi c$. Since impedance of $S_a = -iS_a/\omega$, $R_a = S_a Q_a/\omega_a$, and impedance of $M_a = iS_a\omega/\omega_a^2$, it follows that

$$|u_1 - u_2|^2/|u_1|^2 = [1/Q_a^2 + (\omega/\omega_a)^2]/[1/Q_a^2 + (\omega/\omega_a - \omega_a/\omega)^2]. \quad \text{(A3)}$$

This is a function of V only insofar as Q_a depends on it implicitly (see Fig. 3) or above approximation unacceptable.

With f holes closed, power radiated is (1)×(2).

With f holes open, power radiated is (1)×(2)×(3).

Except near resonance, relative variation of radiated power with frequency is

$$[1 - (\omega_b/\omega)^2]^{-2} \cdot [1 - (\omega_a/\omega)^2]^{-2}. \quad \text{(A4)}$$

By use of Eqs. (1)–(3), the ratio of sound pressures produced at the two resonances assumes a simple form:

$$p_b/p_a = (f_b/f_a)^2 \cdot (Q_b/Q_a). \quad \text{(A5)}$$

APPENDIX II

Refer to Fig. 5 and take $R = (SM)^{\frac{1}{2}}/Q$, $\omega_b = (S/M)^{\frac{1}{2}}$, $\bar{Z}_b = Z_b/(SM)^{\frac{1}{2}} = 1/Q + i(\Omega - 1/\Omega)$.

The line over other impedances indicates similar normalizing to $(SM)^{\frac{1}{2}}$.

$$\alpha_1 = 4\pi l_1/l_{bs} \quad \text{and} \quad \alpha_2 = 4\pi l_2/l_{bs}.$$

With this nomenclature, calculations were made as follows:

$$\bar{Z}_1/\bar{K} = Z_1/K = \frac{\bar{Z}_b/\bar{K} + i\tan\alpha_1\Omega}{1 + i(\bar{Z}_b/\bar{K})\tan\alpha_1\Omega}$$

$$\bar{Z}_2/\bar{K} = Z_2/K = -i\cot\alpha_2\Omega.$$

BIBLIOGRAPHY

1) Abbott, R.B. & Purcell, G.H.; Journal of the Acoustical Society of America (JASA), 13, #1, July 1941

2) Abele & Niederheitmann; The Violin, The New Temple Press, London, (no publication date given)

3) Ascherl, J.F.; Industrial Finishing, 43, #3, Feb. 1967

4) Bradley, J.S. & Stewart, T.W.W.; JASA, 48, # 2, pt 2, Aug. 1970

5) Castle, F. Violin Tone-Peculiarities, H.H. Ragon & Sons, 1906

6) Davidson, P.; The Violin, F. Pitman Co. London, 1881

7) Fletcher, H., Blackham, E.D., & Geertsen, O.N.; JASA, 37, #5, May 1965

8) Fletcher, H. & Saunders, L.C.; JASA, 41, #6, June 1967

9) Hopping, A.S., Hutchins, C.M. & Saunders, F.A.; JASA, 32, #11 Nov. 1960

10) Hutchins, C.M.; Scientific American, 207, Nov. 1962

11) Hutchins, C.M.; Physics Today, 20, Feb. 1967

12) Hutchins, C.M. & Fielding, F.L.; Physics Today, 21, July 1968

13) Meinel, H.; Soviet Physics-Acoustics, 6, 1960

14) Meinel, H.; JASA, 29, 1957

15) Ritter, W.; Industrial Finishing, 32, #11, Sept. 1956

16) Saunders, F.A.; Journal of the Franklin Institute, 229, pages 1-20, 1940

17) Saunders, F.A.; JASA, 17, #3, Jan. 1946

18) Saunders, F.A.; JASA, 25, #3, May 1953

19) Schelleng, J.C.; JASA, 35, March 1963

20) Schelleng, J.C.; JASA, 44, #5, Nov. 1968

21) Varga, J.; Industrial Finishing, 23, #10, Aug. 1947

22) Yankovskii, B.A.; Soviet Physics-Acoustics, 11, #3, Jan-March 1966

23) Yankovskii, B.A.; Soviet Physics-Acoustics, 13, #1, July-Sept. 1967